called dissipation.” We speak of a distribution of heat in a body or system of bodies as having motivity, and we may regard it from without or from within the system.

In the first case it expresses the amount of work which can be obtained by means of perfect engines employed to reduce the whole system to some definite temperature, that, say, of the surrounding medium. In the second case the system is regarded as self-contained, its hotter parts acting as sources, and its colder parts as condensers for the perfect engine.

As an instance of internal motivity we may take the case of a system consisting of two equal portions of the same substance at different temperatures, say a pound of boiling water and a pound of ice-cold water. If we neglect the (small) change of specific heat with tempera­ture, it is found that, when the internal motivity of the system is exhausted, the temperature is about 46° C., being the centigrade temperature corresponding to the geo­metrical mean of the original absolute temperatures of the parts. Had the parts been simply mixed so as to dissipate the internal motivity, the resulting temperature would have been 50° C. Thus the work gained *(i.e.,* the original inter­nal motivity) is the equivalent of the heat which would raise two pounds of water from 46° C. to 50° C.

As an instance of motivity regarded from without we may take the simple case of the working substance in § 2, on the hypothesis that there is an assigned lower tempera­ture limit. As there is no supply of heat, it is clear that the maximum of work will be obtained by allowing the substance to expand adiabatically till its temperature sinks to the assigned limit.

Thus if P (fig. 3) be its given position on Watt’s diagram, PQ the adiabatic through P, and P'Q the isothermal of the lower temperature limit, Q is determinate, and the motivity is the area PQNM. If, again, we wish to find the motivity when the initial and final states P and P' are given, with the condition that the temperature is not to fall below that of the state P', the problem is reduced to finding the course PP' for which the area PP'M'M is greatest. As no heat is supplied, the course cannot rise above the adiabatic PQ, and by hypothesis it cannot fall below the isothermal P'Q,—hence it must be the broken line PQP'. Thus, under the circumstances stated, the motivity is represented by the area MPQP'M'. If any other lawful course, such as PP', be taken, there is an un­necessary waste of motivity represented by the area PQP'.

10. Elementary Thermodynamic Relations.—From what precedes it is clear that, when the state of unit mass of the working substance is given by a point in the diagram, an isothermal and an adiabatic can be drawn through that point, and thus φ and t are determinate for each particular substance when p and v are given. Thus any two of the four quantities p, v, t, φ may be regarded as functions of the other two, chosen as independent variables. The change of energy from one state to another can, of course, be expressed as in § 9, above. Thus, putting E for the energy, we have at once dE=tdφ-ρdv . . . . .(1)

if φ and υ be chosen as independent variables, and if heat be measured, as above, in units of work. This equation expresses, in symbols, the two laws of thermodynamics. For it states that the gain of energy is the excess of the heat supplied over the work done, which is an expression of the first law. And it expresses the heat supplied as the product of the absolute temperature by the gain of entropy, which is a statement of the second law in terms of Thomson’s mode of measuring absolute temperature.

But we now have two equations in partial differential co­efficients :—

(*d*E/*d*φ) = t, (dE/dv) = -p .

From these we have two expressions for the value of (d2E/dvdφ)

Equating them, we are led to the thermodynamic relation (dt/dv) = - (dp/dφ) ,

the differential coefficients being again partial.

This expresses a property of all " working substances, ” defined as in § 1. To state it in words, let us multiply and divide the right hand side by t, and it then reads :—

The rate at which the temperature falls off per unit increase of volume in adiabatic expansion is equal to the rate at which the pres­sure increases per dynamical unit of heat supplied at constant volume, multiplied by the absolute temperature.

To obtain a similar result with v and t as independent variables, we have only to subtract from both sides of (1) the complete differential d(tφ), so that

d(E - tφ) = - φdt - pdυ.

Proceeding exactly as before, we find

(dφ/dv) = (dp/dt) .

In words this result runs (when both sides are multiplied by t) :—

The rate of increase of pressure with temperature at constant volume, multiplied by the absolute temperature,, is equal to the rate at which heat must be supplied per unit increase of volume to keep the temperature constant.

Very slight variations of the process just given obtain the follow­ing varieties of expression :—

(*dv*/*d*φ) = (dt/dp) and (dv/dt) = - (dφ/dp) ,

which are to be interpreted as above.

11. Increase of Total Energy under various Conditions.—The expression (1) of § 10 may be put in various forms, each convenient for some special purpose. We give one example, as sufficiently showing the processes employed. Thus, suppose we wish to find how the energy of the working substance varies with its volume when the temperature is kept constant, we must express dE in terms of dν and dt. Thus

dE = t(dφ/dt)dt + t(dφ/dv)dv - pdv.

But we have, by § 10, under present conditions

(dφ/dv) = (dp/dt) .

Hence (dE/dv) = t(dp/dt) - p ,

a result assumed in a previous article (Radiation, vol. xx. p. 217).

If the working substance have the property (that of the so-called “ ideal ” perfect gas)

pv=Rt,

we see that, for it,

(*d*E/*dv*) = 0

The energy of (unit mass of) such a substance thus depends upon its temperature alone.

12. Specific Heat of a Fluid.—Specific heat in its most general acceptation is the heat required, under some given condition, to raise the temperature of unit mass by one degree. Thus it is the heat taken in while the working substance passes, by some assigned path, from one isothermal t to another t+1; and this may, of course, have as many values as there are possible paths. Usually, however, but two of these paths are spoken of, and these are taken parallel respectively to the coordinate axes in Watt’s diagram, so that we speak of the specific heat at constant volume or at con­stant pressure. In what follows these will be denoted by c and k respectively.

Take v and p for the independent variables, as in the diagram, and let κ be the specific heat corresponding to the condition f(v, p) = const.

Then κdt=tdφ = t[(dφ/dv)(dv) + (dφ/dp)(dp)] ;

while 0 = (df/dv)dv + (df/dp)dp ,

and dt = (dt/dv)dv + (dt/dp)dp .

(dφ/dv)(df/dp) - (dφ/dp)(df/dv)

Thus κ = t???sd

This expression vanishes if f and φ vary together, i.e., in adiabatic expansion, and becomes infinite if f and t vary together, i.e., in isothermal expansion ; as might easily have been foreseen. Other­wise it has a finite value. It is usual, however, to choose v and t as independent variables, while we deal analytically (as distin­guished from diagrammatically) with the subject. From this point of view we have