κdt = t(dφ/dv)(dv) + (dφ/dt)(dt) .

But the last term on the right is, by definition, cdt ; so that (κ-c)dt=t(dφ/dt)dv

at

with the condition

(df/dt)dt + (df/dv)dv = 0.

Thus κdt = -t(dφ/dv)(df/dt)/(df/dv)

which is a perfectly general expression. As the most important case, let f represent the pressure, then we see, by § 10, that

dφ/dv = dp/dt ,

and the formula becomes

k - c = -t (dp/dt)2 /(dp/dv)

13. Properties of an Ideal Substance which follows the Laws of Boyle and Charles.—Closely approximate ideas of the thermal behaviour of a gas such as air, at ordinary temperatures aud pres­sures, may be obtained by assuming the relation

pv=Rt,

which expresses the laws of Boyle and Charles. Thus, by the formula of last section, we have at once

k - c = t(R2/v2)/(p/v) = R

a relation given originally by Carnot.

Hence, in such a substance,

dφ = c(dt/t) + (k - c)(dv/v)

or φ - φ0 = clogt + (k - c)logv

In terms of volume and pressure, this is

Φ - φ0= clogp/R + klogv ,

or pvk/c=Rϵ(Φ-Φ0)∕c ,

the equation of the adiabatics on Watt’s diagram.

This is (for φ constant) the relation between p and v in the pro­pagation of sound. It follows from the theory of wave-motion (Hydromechanics) that the speed of sound is

√*k/c*(R*t*) ,

where t is the temperature of the undisturbed air. This expres­sion gives, by comparison with the observed speed of sound, a very accurate determination of the ratio k∣c in terms of R. The value of R is easily obtained by experiment, aud we have just seen that it is equal to k - c ; so that k and c can be found for air with great accuracy by this process,—a most remarkable instance of the indirect measurement of a quantity (c) whose direct determination presents very formidable difficulties.

14. Effect of Pressure on the Melting or Boiling Point of a Sub­stance.—By the second of the thermodynamic relations in § 10, above, we have

(dp/dt) = (dφ/dv) ,

so that

∙-(∣HHHt>∙

But, if the fraction e of the working substance be in one molecular state (say liquid) in which V0 is the volume of unit mass, while the remainder 1 - e is in a state (solid) where V1 is the volume of unit mass, we have obviously

v=eV0 + (1-e)V1.

Let L be the latent heat of the liquid, then

(dφ/dv) = (tdφ)/[t(V0 - V1)de] = L/[t(V0 - V1)] .

Also, as in a mixture of the same substance in two different states, the pressure remains the same while the volume changes at con­stant temperature, we have dp∣dv=0, so that finally -i⅞⅝

which shows how the temperature is altered by a small change of pressure.

In the case of ice and water, V1 is greater than V0, so the temperature of the freezing-point is lowered by increase of pressure. When the proper numerical values of V0, V1, and L are introduced, it is found that the freezing point is lowered by about 0o⋅074 C. for each additional atmosphere.

When water and steam are in equilibrium, we have V0 much greater than V1, so that the boiling-point (as is well known) is raised by pressure. The same happens, and for the same reason, with the melting point, in the case of bodies which expand in the act of melting, such as beeswax, paraffin, cast-iron, and lava. Such bodies may therefore be kept solid by sufficient pressure, even at temperatures far above their ordinary melting points.

This is, in a slightly altered form, the reasoning of James Thomson, alluded to above as one of the first striking applications of Carnot’s methods made after his work was recalled to notice.

15. Effect of Pressure on Maximum Density Point of Water.— One of the most singular properties of water at atmospheric pres­sure is that it has its maximum density at 4° C. Another, first pointed out by Canton in 1764, is that its compressibility (per atmosphere) is greater at low than at ordinary temperatures,—being, according to his measurements, 0⋅000,049 at 34o F., and only 0⋅000,044 at 64o F. It is easy to see (though it appears to have been first pointed out by Puschl in 1875) that the second of these properties involves the lowering of the maximum density point by increase of pressure. To calculate the numerical amount of this effect, note that the expansibility, like all other thermal properties, may be expressed as a function of any two of the quantities p, v, t, Φ ; say in the present case p and t. Then we have for the expan­sibility

e-K⅛)^('⅛)log,,^zfei,∙

Also the compressibility may be expressed as C = -1/v(dv/dp) = -(d/dp)logv

The relation between small simultaneous increments of pressure and temperature, which are such as to leave the expansibility unchanged, is thus

(S>+(∣>-°∙

Now the expansibility is zero at the maximum density point, for which therefore this equation holds. But the equations above give

(∣)-(⅛>-(£h

so that φ-(>)\*-∙∙

The volume of water at low temperatures under atmospheric pres­sure varies approximately as

144,000

Thus we have (de/dt) = 1/72,000 nearly; and from Canton’s experi­mental result above stated we gather that (roughly at least)

-0 000,005^ = -0 000,000,3 ;

from which the formula gives - 0°⋅02 C. nearly for the change of the maximum density point due to one additional atmosphere.

Recent investigations, carried out by direct as well as by indirect methods, seem to agree in showing that the true value is somewhat less than this, viz., about -0o∙018 C. ; so that water has its maximum density at 0° C. when subjected to about 223 atmo­spheres. Thus, taking account of the result of § 14 above, we find that the maximum density point coincides with the freezing point at - 2o∙8 C. under an additional pressure of about 377 atmospheres, or (say) 2⋅5 tons weight per square inch.

16. Motivity and Entropy, Dissipation of Energy.—The motivity of the quantity H of heat, in a body at temperature t, is

H(t-t0)∕t ,

where t0 is the lowest available temperature.

The entropy is expressed simply as

H∕t,

being independent of any limit of temperature.

If the heat pass, by conduction, to a body of temperature t' (less than t, but greater than t0), the change of motivity (i.e., the dis­sipation of energy) is

H*t*0(1/*t* - 1/*t*') ,

which is, of course loss; while the corresponding change of entropy is the gain

H(1/t' - 1/t) .

The numerical values of these quantities differ by the factor t0, so that, if we could have a condenser at absolute zero, there could be no dissipation of energy. But we see that Clausius’s statement that the entropy of the universe tends to a maximum is practically merely another way of expressing Thomson’s earlier theory of the dissipation of energy.

When heat is exchanged among a number of bodies, part of it