the range of spring tide will be about three times that of neap tide.

So far we have supposed the luminaries to move on the equator ; now let us consider the case where the moon is not on the equator. It is clear in this case that at any place the moon’s zenith distance at the upper transit is different from her nadir distance at the lower transit. But the tide-generating force is greater the smaller the zenith or nadir distance, and therefore the forces are different at successive transits. This was not the case when the moon was deemed to move on the equator. Thus there is a tendency for two successive lunar tides to be of unequal heights, and the resulting inequality of height is called a “diurnal tide.” This tendency vanishes when the moon is on the equator ; and, as this occurs each fortnight, the lunar diurnal tide is evanescent once a fort­night. Similarly in summer and winter the successive solar tides are generally of unequal height, whilst in spring and autumn this difference is inconspicuous.

One of the most remarkable conclusions of Laplace’s theory of the tides, on a globe covered with ocean to a uniform depth, is that the diurnal tide is everywhere non­existent. But this hypothesis differs much from the reality, and in fact at some ports the diurnal tide is so large that during two portions of each lunation there is only one great high water and one great low water in each twenty- four hours, whilst in other parts of the lunation the usual semi-diurnal tide is observed.

§ 4. *Historical Sketch.@@1*

In 1687 Newton laid the foundation for all that has since been added to the theory of the tides when he brought his grand generalization of universal gravitation to bear on the subject. Kepler had indeed at an earlier date recognized the tendency of the water of the ocean to move towards the centres of the sun and moon, but he was unable to submit his theory to calculation. Galileo expresses his regret that so acute a man as Kepler should have produced a theory which appeared to him to rein­troduce the occult qualities of the ancient philosophers. His own explanation referred the phenomenon to the rota­tion and orbital motion of the earth, and he considered that it afforded a principal proof of the Copernican system.

In the 19th corollary of the 66th proposition of book i. of the *Principia,* Newton introduces the conception of a canal circling the earth, and he considers the influence of a satellite on the water in the canal. He remarks that the movement of each molecule of fluid must be accelerated in the conjunction and opposition of the satellite with the molecule, and retarded in the quadratures, so that the fluid must undergo a tidal oscillation. It is, however, in propositions 26 and 27 of book iii. that he first deter­mines the tidal force due to the sun and moon. The sea is here supposed to cover the whole earth, and to assume at each instant a figure of equilibrium, and the tide-gener­ating bodies are supposed to move in the equator. Con­sidering only the action of the sun, he assumes that the figure is an ellipsoid of revolution with its major axis directed towards the sun, and he determines the ellipticity of such an ellipsoid. High solar tide then occurs at noon and midnight, and low tide at sunrise and sunset. The action of the moon produces a similar ellipsoid, but of greater ellipticity. The superposition of these ellipsoids gives the principal variations of tide. He then proceeds to consider the influence of latitude on the height of tide, and to discuss other peculiarities of the phenomenon. Observation shows, however, that spring tides occur a day and a half after syzygies, and Newton falsely attributed

this to the fact that the oscillations would last for some time if the attractions of the two bodies were to cease.

The Newtonian hypothesis, although it fails in the form which he gave to it, may still be made to represent the tides, if the lunar and solar ellipsoids have their major axes always directed towards a fictitious moon and sun, which are respectively at constant distances from the true bodies ; these distances are such that the syzygies of the fictitious planets occur about a day or a day and a half later than the true syzygies. In fact, the actual tides may be supposed to be generated directly by the action of the real sun and moon, and the wave may be imagined to take a day and a half to arrive at the port of observation. This period has accordingly been called “ the age of the tide.” In what precedes the planets have been supposed to move in the equator ; but the theory of the two ellipsoids cannot be reconciled with the truth when they move in orbits inclined to the equator. At equatorial ports the theory of the ellipsoids would af spring tides give morn­ing and evening high waters of nearly equal height, what­ever the declinations of the bodies. But at a port in any other latitude these high waters would be of very different heights, and at Brest, for example, when the declinations of the bodies are equal to the obliquity of the elliptic, the evening tide would be eight times as great as the morning tide. Now observation shows that at this port the two tides are nearly equal to one another, and that their greatest difference is not a thirtieth of their sum.

Newton here also offered an erroneous explanation of the phenomenon. In fact, we shall see that by Laplace’s dynamical theory the diurnal tide is evanescent when the ocean is of uniform depth over the earth. At many non­European ports, however, the diurnal tide is very important, and thus as an actual means of prediction the dynamical theory, where the ocean is treated as of uniform depth, may be hardly better than the equilibrium theory.

In 1738 the Academy of Sciences of Paris offered, as a subject for a prize, the theory of the tides. The authors of four essays received prizes, viz., Daniel Bernoulli, Euler, Maclaurin, and Cavalieri. The first three adopted not only the theory of gravitation but also Newton’s method of the superposition of the two ellipsoids. Bernoulli’s essay contained an extended development of the conception of the two ellipsoids, and, under the name of the equilibrium theory, it is commonly associated with his name. Laplace gives an account and critique of the essays of Bernoulli and Euler in the *Mécanique Celeste.* The essay of Mac­laurin presented little that was new in tidal theory, but is notable as containing those theorems concerning the attraction of ellipsoids which we now know by his name. In 1746 D’Alembert wrote a paper in which he treated the tides of the atmosphere; but this work, like Maclaurin’s, is chiefly remarkable for the importance of collateral points.

The theory of the tidal movements of an ocean was therefore, as Laplace remarks, almost untouched when in 1774 he first undertook the subject. In the *Mécanique Céleste* he gives an interesting account of the manner in which he was led to attack the problem. We shall give below the investigation of the tides of an ocean covering the whole earth; the theory is substantially Laplace’s, although presented in a somewhat different form. This theory, although very wide, is far from representing the tides of our ports. Observation shows, in fact, that the irregular distribution of land and water and the variable depth of the ocean produce an irregularity in the oscilla­tions of the sea of such complexity that the rigorous solu­tion of the problem is altogether beyond the power of analysis. Laplace, however, rested his discussion of tidal observation on this principle—*The state of oscillation of a system of bodies in which the primitive conditions of moνe-*

@@@1 Founded on Laplace, Mécanique Céleste, bk. xiii. chap. i.