west axis is proportional to sin 2δ sin λ. Obviously the latter is circular in latitude 30o. When the moon is on the equator, the maximum deflexion occurs when the moon’s local hour-angle is 45°, and is then equal to

3∞ ∕ α∖3 .

— ( - 1 cos λ.

2Jf∖r∕

At Cambridge in latitude 520 43' this angle is 0"⋅0216.

An attempt, made by George and Horace Darwin,1 to measure the lunar deflexion of a pendulum failed on account of incessant variability of level occurring in the supports of the pendulum and arising from unknown terrestrial changes. The work done, therefore, was of no avail for the purposes for which it was instituted, but remained as a contribution to an interesting subject now be­ginning to be studied, viz., the small changes which are always taking place on the upper strata of the earth.

§ 9. Correction to Equilibrium Theory for Continents.

In the equilibrium theory as worked out by Newton and Ber­noulli it is assumed that the figure of the ocean is at each instant one of equilibrium under the action of gravity and of the tide­generating forces. Sir W. Thomson has, however, reasserted@@2 a point which was known to Bernoulli, but has since been overlooked, namely, that this law of rise and fall of water cannot, when por­tions of the globe are continents, be satisfied by a constant volume of water in the ocean. The law would still hold if water were appropriately supplied to and exhausted from the ocean ; and, if in any configuration of the tide-generating body we imagine water to be instantaneously so supplied or exhausted, the level will every­where rise or fall by the same height. Now the amount of that rise or fall depends on the position of the tide-generating body with reference to the continents, and is different for each such position. Conversely, when the volume of the ocean remains con­stant, we have to correct Bernoulli’s simple equilibrium theory by an amount which is constant all over the globe at any instant, but which changes in time. Thomson’s solution of this problem has since been reduced to a form which is easier to grasp intelligently than in the shape in which he gave it, and the results have also been reduced to numbers.3 It appears that there are four points on the earth’s surface at which in the corrected theory the semi­diurnal tide is evanescent, and four others where it is doubled. A similar statement holds for the diurnal tide. As to the tides of long period, there are two parallels of latitude of evanescent and two of doubled tide.

Now in Bernoulli's theory the semi-diurnal tide vanishes at the poles, the diurnal tide at the poles and the equator, and the tides of long period in latitudes 35o 16' north and south. The numerical solution of the corrected theory shows that the points and lines of doubling and evanescence in every case fall close to the points and lines where in the uncorrected theory there is evanescence. When in passing from the uncorrected to the corrected theory we speak of a doubled tide, the tide doubled may be itself nil, so that the result may still be nil. The conclusion, therefore, is that Thomson’s cor­rection, although theoretically interesting, is practically so small that it may be left out of consideration.

III. Dynamical Theory of Tides.

§ 10. Historical Explanation.

The problem of tidal oscillation is essentially a dynamical one. Even when the ocean is taken as covering the whole earth, it pre­sents formidable difficulties, and this is the only case in which it has been hitherto solved.4 Laplace gives the solution in bks. i. and iv. of the Mécanique Céleste ; but his work is unnecessarily complicated by the inappropriate introduction of spherical harmonic analysis, and it is generally admitted that his investigation is difficult. Airy, in his “Tides and Waves” (in Ency. Metrop.) presents the solution free from that complication, but he has made a criticism of Laplace’s method which we believe to be wrong. Sir W. Thomson has written some interesting papers (in Phil. Mag., 1875) in justification of Laplace, and on these we base the following paragraphs. This portion of the article is given more fully than others, because there exists no complete presentment of the theory free from objections of some kind.

§11. Equations of Motion.

Let r, θ, φ be the radius vector, co-latitude, and east longitude of a point with reference to an origin, a polar axis, and a zero-meridian rotating with a uniform angular velocity n from west to east. Then, if R, H, S be the radial, co-latitudinal, and longitudinal accelerations of the point, we have

-S√⅛7-"<M 1

≡=ia(,sS) →≡"∞><,(^+√ 1 <15,∙

5'=?5Ε»£’’Λ1\*ι'(ώ+”)] .

Now suppose that the point never moves far from a zero position and that its displacements ξ, ηsinθ co-latitudinally and longi­tudinally are very large compared with its radial displacement p, and that the velocities are so small that their squares and products are negligible compared with n2r2 ; then we have

a very small quantity ;

. οdφ d. . a.

rsιnθdt=dt^ainθ^ dθ~dξ r dt~ dt'

Hence (15) is approximately

R= -τι2rsin20 >

S=g-2»sin«oos«|l (le)

2Γ=sin 0 + 2n cos 0 ~J

With regard to the first equation of (16), we observe that the time has disappeared, and that R has exactly the same form as if the system were rendered statical by introducing a potential ½n2r2sin2θ and annulling the rotation of the axes. Since inertia plays no sensible part radially, it follows that, if we apply these expressions to the formation of equations of motion for the ocean, the radial motion need not be considered. We are left, therefore, with only the last two equations of (16).

We now have to consider the forces by which an element of the ocean is urged in the direction of co-latitude and longitude. These forces are those due to the external disturbing forces and to the pressure of the surrounding fluid, the attraction of the fluid on itself being supposed negligible. We have seen in (9) that, if fluid on a sphere of radius a be under the action of disturbing forces whose potential is Ur2, and if r=a + l) be the equation to the sur­face, then must gh = Ua2. Hence, if t be the equilibrium height of tide, the potential of the disturbing force is gtrfd1. But, if the elevation be h, the potential under which it would be in equili­brium is gtyrt∣a!t. Therefore, if h be the elevation of the tide in our dynamical problem, the forces due to hydrostatic pressure on an element of the ocean are the same as would be caused by a potential - g∖ffar. Hence it follows that the whole forces on the element are those due to a potential - <∕(lj - t)ιa∣a2. Therefore from (16) we see that the equations of motion are

g-2≡nfcosS⅛=-⅛(⅛-t) 1

sinS⅛ + 2,1∞s^ =

It remains to find the equation of continuity. This may be deduced geometrically from the consideration that the volume of an element of the fluid remains constant ; but a shorter way is to derive it from the equation of continuity as it occurs in ordinary hydrodynamical investigations. If V be a velocity potential, the equation of con­tinuity for incompressible fluid is

s¾∙(⅛7 sinβδβw) +δ⅛(r staβ ιa5rδ≠) + δ0) = 0∙

dφ∖ r sin 0 dφ ∕

The element referred to in this equation is defined by r, θ, φ, r + δr, θ + δθ, φ + δφ. The co-latitudinal and longitudinal veloci­ties are the same for all the elementary prism defined by θ, φ, θ + δθ, φ+δφ, and the sea bottom. Then ⅛√=⅞ —~~⅞τ =~~

rdθ dt rsva,θdφ sin 4<i "lj- since the radial velocity is d↑)∣dt at the surface of the ocean, where r=a + y, and is zero at the sea bottom, where r=a, we have Hence, integrating with respect to r from

r=a+λ to r=a, and again with respect to t from the time t to the time when lj, ξ, η all vanish, and treating γ and f) as small com­pared with a, we have

ljα sin 0 + ½ff(yξ sin 0) + J~(7R sin 0) = 0 (18).

This is the equation of continuity, and, together with (17), it forms the system which must be integrated in the general problem of the tides. The difficulties in the way of a solution are so great that none has hitherto been found, except on the supposition that y, the depth of the ocean, is only a function of latitude. In this case (18) becomes

@@@1 Reports to the *British Assοc.,* 1881 (York) and 1882 (Southampton).

@@@2 Thomson and Tait, *Nat. Phil.,* § 807.

@@@3 Darwin and Turner, *Proc. Roy. Soc.,* 1886.

@@@4 Sir W. Thomson’s paper “ On the Gravitational Oscillations of Rotating Water,” in *Phil. Mag.,* August 1880, bears on the same subject. It is the only attempt which has hitherto been made to consider the effects of the earth’s rotation on the oscillations of land-locked seas.