Schedule of Lunar Tides.

[A, i.]—Universal Coefficient = (3/2)(m/M)(a/c)3a. Semi-diurnal Tides ; General Coefficient=cos2 λ.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Descrip­tive Name. | Initial | Coefficient. | Mean Value of Coefficient | Argument  2*t*+2(*h-v*.) | Speed in Degrees per m.s. Hour. |
| Principal lunar. | m2 | ½(l-iβ2)cos4½*I* | •45426 | -2(s-ξ) | 28°⋅9841042 |
| Luni-solar (lunar portion). | K2 | ½(l+3/2*e*2)½sin2*I* | •03929 |  | 30o⋅0821372 |
| Larger elliptic. | N | ½∙7/2*e*cos4½*I* | •08796 | -*2(s-ξ)-(s-p*) | 280∙4397296 |
| Smaller  elliptic. @@1 | L | ½⋅½*e*cos4½*I*×  (l-12tn2Zcos(2p-2^)} 1 | •01257 | Γ -2(s-t)+(,-i>)-R+ιr  J where „ . >■>  1 tanR- 6≡1∏2(p-0  V cot2JZ-6cos2(p-0 | 29o∙5284788 |
| Elliptic, second order. | 2N | ½.127-*e*2cos4½*I* | •01173 | -2(s-ξ)-2(*s-p*) | 27°∙8953548 |
| Larger  evectional.@@2 | *V* | }.∙γ⅛mecos4⅛Γ | ⋅01234@@3 •01706 | -2(*s*-ξ+(*s-p*)+2*h*-2*s* | 28o⋅5125830 |
| Smaller evectional. | *∖* | ⅛ ∣jpnecos4⅛r | •001763 •00330 | -*2(s-ξ)-(s-p)-2h+2s+π* | 29o∙4556254 |
| Varia­tional. @@4 |  | i∙¾>m2cos4il | •007363 •01094 | -2(s-ξ)+2*h*-2*s* | 27o⋅9682084 |

[A, ii.]—Diurnal Tides ; General Coefficient = sin 2λ.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Descrip­tive Name. | Initial. | Coefficient | Mean Value of Coefficient | Argument *t+(h-v)∙* | Speed in Degrees per m.s.  Hour. |
| Lunar di­urnal. | 0 | (1 - Se2)⅛ sin *I*cos2⅛I | •18856 | -2(s-ξ)+½π | 13°∙9430356 |
| γ+2σ. Luni-solar  (lunarpor-tion). | OO  K1 | (l-∣β2⅛sinZsin2⅛7  (l+je¾sinicosl | •00812  •18115 | +2(s-9~iJΓ  -½π | 16°∙1391010  15o∙0410686 |
| Larger elliptic. | Q | Ie.∣sinZcos2⅛Γ | •03651 | -2(s-ξ)-(*s-p*)  +⅜1Γ | 13o∙3986609 |
| Smaller  elliptic.@@5 | M1 | β.⅜sinZcos2⅛7×  √ {f+3cos2(p-ξ)} | ⋅00522@@@6  •01649 | √s-i)+Q-Jιτ where tanQ  = ⅛tan(p-f) | 14°∙4920520 |
| γ+σ-ῶ.  Elliptic, second order. | J  γ-4σ+2ῶ | Je·JsinZcosZ  Ve2∙⅜sinΓcos2⅛Z | ⋅01485  ⋅00487 | +(s-p)-½π  -2(s-9-2(3-?) +⅛Γ | 150∙5854433  12o⋅854286: |
| Evectional. | γ-3σ-ῶ+2η | ∣Wnw-i sinlrcos2l *I* | •00512@@7 ⋅00708 | -2(s-ξ)+(*s-p*)  +2h-2s+½π | 13° ∙4715144 |

[A, iii. ]—Long Period Tides ; General Coefficient ½ - 3/2 sin2 λ.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Descrip­tive Name. | Initial. | Coefficient | Mean Value of Coefficient. | Argument. | Speed in De­grees per m.s.  Hour. |
| Change of mean level. |  | (l+?e2)Kl-?sin2I) | •25224@@8 | Of variable part is *N,* the long. of node | 19o⋅34 per annum |
| Monthly. | Mm | 3e⋅⅓(l-3/2sin2*I*) | •04136 | *s-p* | 0°∙5443747 |
| Evectional monthly. | σ-2η+ῶ | Vmβ.⅛(l-3sin2I) | •00580@@9 ⋅00755 | ί *-s-ρ)*  1 +2s-2⅛ | 0°∙4715211 |
| Luni-solar fort­nightly. @@10 | MSf | 3m2⅓(l-3/2sin2*I*) | •004229 •00621 | 2(s-h) | l°∙0158958 |
| Fort­nightly. | Mf | (l-⅞β2)⅜sin2I | •07827 | 2(3-ξ) | 1°∙0980330 |
| Ter- mensual. | 3σ-ῶ | Je.isin2Z | •01516 | j(s-2>)+2(s-ξ) | lo∙6424077  1 |

[B.]—Schedule of Solar Tides.

Solar Tides ; Universal Coefficient*a. 2m∖cJ*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Descriptive Name. | Initial. | Coefficient | Value of Coefficient. | Argu­ment. | Speed in Degrees per m.s.  Hour. |
| [i∙]-S  Principal solar. | emi-  s2 | diurnal Tides ; Gener  ~(1 - Je 2)⅛cos4⅛ω | al Co<  •21137 | jfficient = <  2t | 3OS2λ∙  30°-0000000 |
| Luni-solar (solar por­tion). | K2 | ~'G+K≈)i≡2ω | •01823 | 2t÷2Λ | 30o∙0821372 |
| Larger el­liptic. | T | ^'iJe,cos4Jω | •01243 | 2t-(A-p,) | 290∙9589314 |
| ∏i∙]- | -Dii | ιrnal Tides ; General | Coeffi | cient — sin | ι2λ, |
| Solar diur­nat | P | ί -7'(1 - ∣e,2)⅜ sin *ω* cos2 Jω  T | Ό8775 | i-⅛+iπ | 14o∙9589314 |
| Luni-solar (solar por­tion). | Κι | J y(l+⅞e ,2)Jsinωcosω | ■08407 | t+Λ-i7Γ | 15o∙0410686 |
| [iii.]—Loi | ng I | ,eriod Tides ; General | Coeffi | ιcient=⅜ - | - ⅜ sin2 λ. |
| Semi-an­nual. | 1 Ssa | ⅞(1-je,s)⅛sin2ω | ∣∙03643 | 2⅛ | 0o∙0821372 |

From the fourth columns we see that the coefficients in de- Scale of scending order of magnitude are M2, K1 (both combined), S2, import- O, K1 (lunar), N, P, K1 (solar), K2 (both combined), K2 (lunar), Mf, ance of Q, Mm, K2 (solar), Ssa, v, M1, J, L, T, 2N, μ, OO, 3σ-ῶ, tides. y-3σ-ῶ + 2η, γ- 4σ+ 2σ, σ-2η + ῶ, 2(σ-η), λ.

The tides depending on the fourth power of the moon’s parallax arise from the potential V=^-ρ8 (5/2 cos3 z - 3/2 cos z). They give rise to a small diurnal tide M1, and to a small ter-diurnal tide M3 ; but we shall not give the analytical development.

§ 24. Meteorological Tides, Over-Tides, and Compound Tides.

All tides whose period is an exact multiple or submultiple of a mean solar day, or of a tropical year, are affected by meteorological conditions. Thus all the tides of the principal solar astronomical series S, with speeds y-η, 2(γ-η), 3(γ-η), &c., are subject to more or less meteorological perturbation. An annual inequality in the diurnal meteorological tide S1 will also give rise to a tid γ - 2η, and this will be fused with and indistinguishable from the astro­nomical P ; it will also give rise to a tide with speed y, which will be indistinguishable from the astronomical part of K1. Similarly the astronomical tide K2 may be perturbed by a semi-annual in­equality in the semi-diurnal astronomical tide of speed 2(γ-η). Although the diurnal elliptic tide S1 or γ - η and the semi-annual and annual tides of speeds 2η and η are all probably quite insensible as arising from astronomical causes, yet they have been found of sufficient importance to be considered. The annual and semi­annual tides are of enormous importance in some rivers, representing in fact the yearly flooding in the rainy season. In the reduction of these tides the arguments of the S series are t, 2t, 3t, &c., and of the annual, semi-annual, ter-annual tides h, 2h, 3h. As far as can be foreseen, the magnitudes of these tides are constant from year to year.

We have in § 21 considered the dynamical theory of over-tides. The only tides of this kind in which it has hitherto been thought necessary to represent the change of form in shallow water belong to the principal lunar and principal solar series. Thus, besides the fundamental astronomical tides M2 and S2, the over-tides M4, M6, M8, and S4, S6 have been deduced by harmonic analysis. The height of the fundamental tide M2 varies from year to year, according to the variation in the obliquity of the lunar orbit, and this variability is represented by the coefficient cos4 ½I. It is probable that the variability of M4, M6, M8 will be represented by the square, cube, and fourth power of that coefficient, and theory (§ 21) indicates that we should make the argument of the over-tide a multiple of the argument of the fundamental, with a constant subtracted.

Compound tides have been also considered dynamically in § 21. By combining the speeds of the important tides, it will be found that there is in many cases a compound tide which has itself a speed identical with that of an astronomical or meteorological tide. We thus find that the tides O, K1, Mm, P, M2, Mf, Q, M1, L are liable to perturbation in shallow water. If either or both the component tides are of lunar origin, the height of the compound tide will change from year to year, and will probably vary proportionally to the product of the coefficients of the component tides. For the purpose of properly reducing the numerical value of the compound tides, we require not merely the speed, but also the argument. The following schedule gives the adopted initials, argument, and speed of the principal compound tides. The coefficients are the products of those of the two tides to be compounded.

@@@1 Fused with 2γ-σ+ῶ.

@@@2 *m* is the ratio of the moon’s mean motion to the sun’s.

@@@3 In these three entries the lower number gives the value when the co­efficients of the evection and variation have their full values as derived from lunar theory.

@@@4 Indicated by 2MS as a compound tide (see below, § 24).

@@@5 A fusion of γ *-* σ±ῶ, of which the latter is the tide named.

@@@6 The upper number is the mean value of the coefficient of the tide *y - σ* - ῶ ; the lower applies to the tide M1, compounded from the tides y-σ-ῶ and γ*-σ+ῶ.*

7 The lower number gives the value when the coefficients in the evection have their full value as derived from lunar theory.

@@@8 The mean value of this coefficient is ⅓(l+3/2e2)(l-3/2sin2i)(l-⅞sin2ω)=∙25, and the variable part is approximately-(l+3/2e2) sin*i* cos*i* sinω cosωcos*N*= - ∙0328 cos*N*.

@@@9 The lower of these two numbers gives the value when the coefficients in the evection and variation have their full values as derived from lunar theory.

@@@10 Indicated by MSf as a compound tide.