[C.]—*Schedule of Compound Tides.*

|  |  |  |  |
| --- | --- | --- | --- |
| Initials. | Arguments com­bined. | Speed. | Speed in Degrees per m.s. Hour. |
| MK | M2+K1  M4-0 | 3γ-2σ | 44°⋅0251728 |
| MS | m2+s2 | 4γ - 2σ - *2η* | 58°⋅9841042 |
| MSf | s2-m2 | 2σ-2η | 1o⋅0158958 |
| 2ΜΚ | M2+O m4-k1 S2 +K1 | 3γ-4σ  3γ-2η | 42o∙9271398  45o∙0410686 |
| MN | m2+n  S2 +O  S2 -O | 4γ-5σ+ῶ  3γ-2σ-2η  γ+2σ-2η | 57°⋅4238338  43"∙9430356  16°⋅0569644 |
| 2SM | s4-m2 M2+S4 | *2γ+2σ-+4η*  *6γ-2σ-4η* | 31°∙0158958  88°∙9841042 |
| 2MS | m4-s2  M4+S2 | *2γ-4σ+2η*  *6γ-4σ-2η* | 27°∙9682084  87o∙9682084 |

§ 25. On the Form of Presentation of Results of Tidal Observations.

Supposing n to be the speed of any tide in degrees per mean solar hour, and t to be mean solar time elapsing since 0h of the first day of (say) a year of continuous observation, then the immediate result of harmonic analysis is to obtain A and B, two heights (estimated in feet and tenths) such that the height of this tide at the time t is given by A cos nt + B sin nt. If we put R=√(A2 + B2) and tan ϛ= B∕A, then the tide is represented by

R cos (nt - ϛ).

In this form R is the semi-range of the tide in British feet, and ζ is an angle such that ζ∣n is the time elapsing after 0h of the first day until it is high water of this particular tide. It is obvious that ζ may have any value from 0o to 360o, and that the results of the analysis of successive years of observation will not be com­parable with one another when presented in this form.

But let us suppose that the results of the analysis are presented in a number of terms of the form

fHcos(V+u- κ),

where V is a linear function of the moon’s and sun’s mean longi­tudes, the mean longitude of the moon’s and sun’s perigees, and the local mean solar time at the place of observation, reduced to angle at 15o per hour. V increases uniformly with the time, and its rate of increase per mean solar hour is the n of the first method, and is called the speed of the tide. It is supposed that u stands for a certain function of the longitude of the node of the lunar orbit at an epoch half a year later than 0h of the first day. Strictly speaking, u should be taken as the same function of the longitude of the moon’s node, varying as the node moves ; but, as the varia­tion is but small in the course of a year, u may be treated as a constant and put equal to an average value for the year, which average value is taken as the true value of u at exactly mid year. Together V+u constitute that function which has been tabulated as the “argument” in the schedules of § 23. Since V+u are to­gether the whole argument according to the equilibrium theory of tides, with sea covering the whole earth, it follows that κ∕n is the lagging of the tide which arises from kinetic action, friction of the water, imperfect elasticity of the earth, and the distribution of land. It is supposed that H is the mean value in British feet of the semi-range of the particular tide in question ; f is a numerical factor of augmentation or diminution, due to the variability of the obliquity of the lunar orbit. The value of f is the ratio of the “ coefficient ” in the third column of the preceding schedules to the mean value of the same term. For example, for all the solar tides f is unity, and for the principal lunar tide M2 it is equal to cos4 ½I÷cos4 ⅜ω cos4 ½i ; for the mean value of this term has a coefficient cos4½ω cos4½i. It is obvious, then, that, if the tidal observations are consistent from year to year, H and κ should come out the same from each year’s reductions. It is only when the results are presented in such a form as this that it will be possible to judge whether the harmonic analysis is yielding satisfactory results. This mode of giving the tidal results is also essential for the use of a tide-predicting machine (see § 38).

We must now show how to determine H and κ from R and ζ. It is clear that H = R∕f, and the determination of f from the schedules depends on the evaluation of the mean value of each of the terms in the schedules, into which we shall not enter. If V0 be the value of V at 0h of the first day, then clearly

-ζ= V0 + u-κ, so that K=ζ+V0+u.

Thus the rule for the determination of κ is : Add to the value of ζ the value of the argument at 0h of the first day.

The results of harmonic analysis are usually tabulated by giving Η, κ under the initial letter of each tide ; the results are thus comparable from year to year.@@1 For the purpose of using the tide­predicting machine the process of determining H and κ from R and

ζ has simply to be reversed, with the difference that the instant of time to which to refer the argument is 0h of the first day of the new year, and we must take note of the different value of u and f for the new year. Tables@@2 have been computed for f and u for all longitudes of the moon’s node and for each kind of tide, and the mean longitudes of moon, sun, and lunar perigee may be ex­tracted from any ephemeris. Thus when the mean semi-range H and retardation κ of any tide are known its height may be com­puted for any instant. The sum of the heights for all the principal tides of course gives the actual height of water.

§ 26. Numerical Harmonic Analysis for Tides of Short Period.

The tide-gauge (described below, § 36) furnishes us with a con­tinuous graphical record of the height of the water above some known datum mark for every instant of time. The first operation performed on the tidal record is the measurement in feet and deci­mals of the height of water above the datum at every mean solar hour. The period chosen for analysis is about one year and the first measurement corresponds to noon.

If T be the period of any one of the diurnal tides, or the double period of any one of the semi-diurnal tides, it approximates more or less nearly to 24 m. s. hours, and, if we divide it into twenty- four equal parts, we may speak of each as a T-hour. We shall for brevity refer to mean solar time as S-time. Suppose, now, that we have two clocks, each marked with 360o, or 24 hours, and that the hand of the first, or S-clock, goes round once in 24 S-hours, and that of the second, or T-clock, goes round once in twenty-four T-hours, and suppose that the two clocks are started at 0o or 0h at noon of the initial day. For the sake of distinctness, let us imagine that a T-hour is longer than an S-hour, so that the T-clock goes slower than the S-clock. The measurements of the tide curve give us the height of water exactly at each S-hour ; and it is re­quired from these data to determine the height of water at each T-hour. For this end we are, in fact, instructed to count T-time, but are only allowed to do so by reference to S-time, and, moreover, the time is always to be specified as an integral number of hours. Commencing with 0h of the first day, we begin counting 0, I, 2, &c., as the T-hand comes up to its hour-marks. But, as the S-hand gains on the T-hand, there will come a time when, the T-hand being exactly at the p hour-mark, the S-hand is nearly as far as p + ½. When, however, the T-hand has advanced to the p+1 hour- mark, the S-hand will be a little beyond p +1 + ½,—that is to say, a little less than half an hour before p + 2. Counting, then, in T-time by reference to S-time, we jump from p to p + 2. The counting will go on continuously for a number of hours nearly equal to 2p, and then another number will be dropped, and so on throughout the whole year. If it had been the T-hand which went faster than the S-hand, it is obvious that one number would be repeated at two successive hours instead of one being dropped. We may describe each such process as a “change.”

Now, if we have a sheet marked for entry of heights of water according to T-hours from results measured at S-hours, we must enter the S-measurements continuously up to p, and we then come to a change ; dropping one of the S-series, we go on again continu­ously until another change, when another is dropped ; and so on. Since a change occurs at the time when a T-hour falls almost exactly half-way between two S-hours, it will be more accurate at a change to insert the two S-entries which fall on each side of the truth. If this be done the whole of the S series of measure­ments is entered on the T-sheet. Similarly, if it be the T-hand which goes faster than the S-hand, we may leave a gap in the T-series instead of duplicating an entry. For the analysis of the T-tide there is therefore prepared a sheet arranged in rows and columns ; each row corresponds to one T-day, and the columns are marked 0h, 1h,... 23h ; the 0’s may be called T-noons. A dot is put in each space for entry, and where there is a change two dots are put if there is to be a double entry, and a bar if there is to be no entry.@@3 The numbers entered in each column are summed ; the results are then divided, each by the proper divisor for its column, and thus the mean value for that column is obtained. In this way 24 numbers are found which give the mean height of water at each of the 24 special hours. It is obvious that if this process were con­tinued over a very long time we should in the end extract the tide under analysis from amongst all the others ; but, as the process only extends over about a year, the elimination of the others is not quite complete. The elimination of the effects of the other tides may be improved by choosing the period for analysis not exactly equal to one year.

Let us now return to our general notation, and consider the 24 mean values, each pertaining to the 24 T-hours. We suppose that all the tides except the T-tide are adequately eliminated, and, in fact, a computation of the necessary corrections for the absence of complete elimination, which is given in the Tidal Report to the British Association in 1872. shows that this is the case. It is

@@@1 See, for example, a collection of results by Baird and Darwin, *Proc. Roy. Soc.,* No. 23d, 1885.

@@@2 *Report on Harmonic Analysis to Brit. Assoc.,* 1883, and more extended table in Baird’s *Manual of Tidal Observation,* London, 1887.

@@@3 A sample page is given in the *Report* to the Brit. Assoc., 1883.