obvious that any one of the 24 values does not give the true height of the T-tide at that T-hour, but gives the average height of the water, as due to the T-tide, estimated over half a T-hour before and half a T-hour after that hour. A consideration of this point shows that certain augmenting factors, differing slightly from unity, must be applied. In the reduction of the S-series of tides, the numbers treated are the actual heights of the water exactly at the S-hours, and therefore no augmenting factor is requisite.

We must now explain how the harmonic analysis, which the use of these factors presupposes, is carried out

If t denotes T-time expressed in T-hours, and n is 15o, we express the height h, as given by the averaging process above explained, by the formula

h=A0 + A1 cosnt + B1sinnt + A2 cos 2nt + B2sin 2nt + ..., where t is 0, 1, 2, .. . 23. Then, if Σ denotes summation of the series of 24 terms found by attributing to t its 24 values, it is obvious that

A0 = 1/24∑h ; A1 — 1/12∑hcos nt ; B1= 1/12Σhsin nt ; A2 = 1/12 ∑hcos 2nt ; B2 = 1/12 ∑hsin 2nt ; &c., &c.

Since n is 15o and t is an integer, it follows that all the cosines and sines involved in these series are equal to one of the following, viz., 0, ±

sin 15°, ±sin 30o, ± sin 45o, ±sin 60o, ±sin 75o, ±1. It is found convenient to denote these sines by 0, ±S1,±S2,÷S3,±S4, ±S5, ±

1. The multiplication of the 24 h’s by the various S’s and the subsequent additions may be arranged in a very neat tabular form, like that given in a Report to the British Association in 1883. The A’s and B’s having been thus deduced, we have R= √(A2 + B2). R must then be multiplied by the augmenting factor. We thus have the augmented R. Next the angle whose tangent is B/A gives ζ. The addition to ζ of the appropriate V0 + u gives κ, and the multiplication of R by the appropriate 1/f gives H. The reduction is then complete. An actual numerical example of harmonic analysis is given in the Admiralty Scientific Manual (1886) In the article “ Tides ” ; but the process there employed is slightly different from the above, because the series of observations is sup­posed to be a short one.

§ 27. Harmonic Analysis for Tides of Long Period.

For the purpose of determining the tides of long period we have to eliminate the oscillations of water-level arising from the tides of short period. As the quickest of these tides has a period of many days, the height of mean water at one instant for each day gives sufficient data. Thus there will in a year’s observations be 365 heights to be submitted to harmonic analysis. To find the daily mean for any day we take the arithmetic mean of 24 consecutive hourly values, beginning with the height at noon. This height will then apply to the middle instant of the period from 0h to 23h, —that is to say, to 11h 30m at night. The formation of a daily mean does not obliterate the tidal oscillations of short period, be­cause none of the tides, except those of the principal solar series, have commensurable periods in mean solar time. A small correc­tion, or “clearance of the daily mean,” has therefore to be applied for all the important tides of short period, except for the solar tides. Passing by this clearance, we next take the 365 daily means, and find their mean value. This gives the mean height of water for the year. We next subtract the mean height from each of the 365 values, and find 365 quantities δh, giving the daily height of water above the mean height. These quantities are to be the subject of the harmonic analysis, and the tides chosen for evaluation are those which have been denoted above as Mm, Mf, MSf, Sa, Ssa.

Let δh = A cos (σ-ῶ)t + B sin (σ-ῶ)t

+ C cos 2σt + D sin 2σt

+ C'cos2(σ-η)t + D'sin2(σ-η)t (73),

+ E cos ηt + F sin ηt

+ G cos 2ηt +H sin2ηt

where t is time measured from the first 11h 30m. If we multiply the 365 δh's by 365 values of cos (σ-ῶ)t and effect the summation, the coefficients of B,C,D, &c., are very small, and that of A is nearly 182½. Similarly, multiplying by sin (σ - ῶ)t, cos 2σt, &c., we obtain 10 equations for A,B,C, &c., in each of which one coefficient is nearly 182½ and the rest small. These equations are easily solved by successive approximation. In this way A,B,C, &c., are found, and afterwards the clearance to which we have alluded is applied. Finally the cleared A,B,C, &c., are treated exactly as were the components of the tides of short period. Special forms and tables have been prepared for facilitating these operations.

V. Synthetic Method.

§ 28. On the Method and Notation.

The general nature of the synthetic method has been already explained ; we now propose to develop the expressions for the tide from the result as expressed in the harmonic notation. If it should be desired to make a comparison of the results of tidal observation as expressed in the synthetic method with those of the harmonic method, or the converse, or to compute a tide-table from the har­monic constants by reference to the moon’s transits and from the declinations and parallaxes of sun and moon, the analytical ex­pressions of the following sections are necessary.

In chapter iv. the mean semi-range and angle of retardation or lag of any one of the tides have been denoted by H and κ. We shall here, however, require to introduce several of the H’s and κ’s into the same expression, and they must therefore be distinguished from one another. This may in general be conveniently done by writing as a subscript letter the initial of the corresponding tide ; for example Hm, κm will be taken to denote the H and κ of the principal lunar tide M2. This notation does not suit the K2 and K1 tides, and we shall therefore write H", κ" for the semi-diurnal K2, and H', κ' for the diurnal K1 tide. These two tides proceed according to sidereal time and arise from the sun and moon jointly, and a synthesis of the two parts of each is effected in the harmonic method, although that synthesis is not explained in chapter iv. The ratio of the solar to the lunar part of the total K2 tide is ⋅46407 ; hence ⋅683 H" is the lunar portion of the total K2. There will be no occasion to separate the two portions of K1, and we shall retain the synthesis which is effected in the harmonic method.

§ 29. Semi-Diurnal Tides.

The process adopted is to replace the mean longitudes and ele­ments of the orbit in each term of the harmonic development of the schedules of § 23 by hour-angles, declinations, and parallaxes.

At the time t (mean solar time of port reduced to angle) let a, δ, ψ be 〗’s R.A., declination, and hour-angle, and l 〗’s longitude measured from the " intersection. ” These and other symbols when written with subscript accent are to apply to the sun. Then v being the R.A. of the intersection, we have from the right-angled spherical triangle of which the sides are l, δ, a - v the relation tan(α-v)=cosItanl, sin δ=sinIsinl (74).

Now s-ξ is the 〗’s mean longitude measured from the intersection and s-p is the mean anomaly ; hence approximately

l=s-ξ+2esin(s-ρ) (75).

From (74) and (75) we have approximately

a=s+(v - ξ) + 2e sin (s -p) - tan2 ½I sin 2(s - ξ).

Now, h being the ⌾’s mean longitude, t+h is the sidereal hour- angle, and ψ=t + h-a.

Hence

t + h-s-(v-ξ) = ψ + 2esin(s-p)-tan2½Isin2(s-ξ) (76). Again, if we put

cos2∆=1 - ½ sin2 I (77),

we have approximately from (74) and (75)

cos2δ - cos2∆ n, λ

sin2∆ ^c°s & [ r7in

sinδcosδ<Zδ . n. t. f ∖∙°h

whence -7≡- S=sm2<s-flJ

Obviously Δ is such a declination that sin2 Δ is the mean value of sin2 δ during a lunar month. Again, if P be the ratio of the 〗’s parallax to her mean parallax, the equation to the ellipse described gives 1/e(-P- l)=cos (s-p) Ί

t 1 dP . , , 1 (?»

Now it appears in schedule A of § 23 that the arguments of all the lunar semi-diurnal tides are of the form 2(t + h-v)±2(s-ξ) or ±(s-p). It is clear, therefore, that the cosines of such angles may by the relations (76), (78), (79) be expressed in terms of hour-angles, declinations, and parallaxes. Also by means of (77) we may intro­duce Δ in place of I in the coefficients of each term. An approxi­mate formula for Δ is 16o⋅51 + 3o∙44 cos N-0o⋅19 cos 2N. In the Report to the British Association for 1885, the details of the processes indicated are given.

Before giving the formula it must be remarked that the result is expressed more succinctly by the introduction of the symbol δ' to denote the 〗’s declination at a time earlier than that of observa­tion by an interval which may be called the “age of the declina- tional inequality,” and is computed from the formula tan (κ"- κm)∕2σ or 52h-2tan (κ"- κm). Similarly, it is convenient to introduce P'to denote the value of P at a time earlier than that of observation by the “age of the parallactic inequality,” to be computed from tan (κm-κn)∕(σ- ῶ) or 105h⋅3 tan (κm-κn). These two “ages” probably do not differ in general much from a third period, com­puted from (κ8- κm)∕2(σ- η), which is called the “age of the tide.”

The similar series of transformations when applied to the solar tides leads to simpler results, because Δ, is a constant, being 16°∙33, and the “ ages ” may be treated as zero ; besides the terms depend­ing on dδ,/dt and dP,/dt are negligible. If now we denote by h2 the height of water with reference to mean water-mark, in so far as the height is affected by the harmonic tides M2, S2, K2, N, L, T, R,@@1 the harmonic expression is transformed into

@@@1 R is the smaller solar elliptic tide bearing the same relation to T that L does to N amongst the lunar tides. It was omitted as unimportant in schedule [B, i.] of §23.