h2 = h"∙ cos (2≠ - \*m) + H· cos (2≠∕ ~ r\*)

~~cos~~~~2~~~~δ'-cos~~~~2~~~~∆ ,~~~~683 h~~~~,,~~ ~~cos~~ ~~\_~~

sιn2∆, ~~cos~~~~2~~ ~~δ - cosL~~Δ, \_317 H„ cos (2 \_

τ sιn2∆, τ' '

sin δ cos δ ciδ[- ,683H" rr , κ -] .

-√≡P∆7sL~~eos(«·-».j~~^ H-un,∙1J≡<2≠-«J ~~+~~ ~~^~~~~(F~~~~\_~~~~1~~~~)H.M.\*.-~~~~H~~~~·oos»~~~~i~~

cos2∆z ecose x r ,

+ (P,-l⅛⅛os(2≠4→4)

∙∕

cos2 Δ dP∣dl r.\_\_ Hn H, Ί . ,o. .,oλ.

+ecos2∆, (σ-cτ)L ",e~cos(κra-κβ) cosx1-kto)Js1q^ where ϵ is an auxiliary angle defined by

Hn sin κ - Hz sin κi ,β,1

tane = τ~ n ∙ττt (81).

Hn cos κn - H1 cos κι ' '

The first two terms are the principal tides, and the physical origin of the remaining small terms is indicated by their involving δ', δ,, dδ∣dt, P, Pl, dP∣dt The terms in dδ∕dt and dP∣dt are generally smaller than the others.

The approximation may easily be carried further. But the above is in some respects a closer approximation than the expression from which it is derived, since the hour-angles, declinations, and paral­laxes necessarily involve all the lunar and solar inequalities.

§ 30. Syntheses of Solar and of Lunar Portions of the Semi-Diurnal Tide.

Let us write

,r cos2∆ tt . cos2 δ'- cos2 Δ λoλttw , « .

M = —5-r- Hm + r-δ-i ∙683 H cos (c\* - κm)

cos2∆, m sιn2∆, v '

ι cos2 Δ . Hn cos κu - Hi cos κι . .

COS2 Δ, ' e COS e ' , ’

ι cos2δ'-cos2∆ ,jooττw . , „ .

e-μ=κmΛ τ-ζ- · 683 H\*sιn (√ - kto)

Sl∏ ∆z

+⅛⅜∙ (J.-1) ⅛⅛⅛i5mi,.1,i cos2∆, x , ecose v '

sin δcos δ dδ Γ ∙683H" ττ , , . "Ί

-√3P∆7 ώ L~~cosy-«.)"~~ , n -J + co≡2δ. ⅛Γrn c Hn ¾ Ί .

e cos2 ∆y σ - τs |\_ m cos (κm - κn) ~ cos (xl - κm) J ’

M, = Hi + c03a^n^°32^ ∙317H"+(P,-1) ⅛≡⅛

⅛\*l=∙c. '■ '. (82).

Since observation and theory agree in showing that κ" is generally very nearly equal to κs, we are justified in substituting κ, for κ" in the small solar declinational term of (80) involving ⋅317 H". Then, using (82) in (80),

h2=M cos 2(ψ - μ) + M, cos 2(ψ, - μ,) (83).

If the equilibrium theory of tides were true, each H would be pro­portional to the corresponding term in the harmonically developed potential. This proportionality holds nearly between tides of almost the same speed ; hence, using the expressions in the column of co­efficients in schedule [B, i.], § 23 (with the additional tide R there omitted, but having a coefficient (τ,∕τ)½.½e, cos4½ω, found by sym­metry with the lunar tide L), and introducing Δ, in place of ω in the solar tides, we may assume the truth of the proportion

With this assumption, M, reduces to m-S⅜≡'+3^-1)H<=S⅛≡^ + 3(Λ-1)]1

Hence M,=P,s-t⅛H1 (84).

' cos2∆, ' 7

This is the law which we should have derived directly from the equilibrium theory, with the hypothesis that all solar semi-diurnal tides suffer nearly equal retardation. Save for meteorological influ­ences, this must certainly be true.

A similar synthesis of M cannot be carried out, because the con­siderable diversity of speed amongst the lunar tides makes a similar appeal to the equilibrium theory incorrect. It may be seen, how­ever, that it would be more correct to write cos2 δ' instead of cos2 Δ in the coefficient of the parallactic terms in M and 2μ.

The three terms of M in (82) give the height of lunar tide with its declinational and parallactic corrections, and similarly the formula for μ in (82) gives its value and corrections.

If now τ denotes the mean solar time elapsing since the moon’s upper transit and γ the angular velocity of the earth’s rotation, it is clear that the moon’s hour-angle

ψ=(γ~da∕dt)τ; and, since Mcos2(ψ-μ) is a maximum when φ=μ or differs from μ by 180o, it follows that μ∣(γ-da∣dc) is the “interval” from the moon’s upper or lower transit to high water of the lunar tide. Since τ is necessarily less than 12h, we may during the interval from transit to high water take as an approximation da∣dt=σ, the moon’s mean motion.@@1 Hence that interval is μ∣(γ — σ), or 2/29μ hours nearly, when μ is expressed in degrees. Thus (82) for μ gives by its first term the mean interval for the lunar tide, and by the subsequent terms the declinational and parallactic corrections.

We have said that the synthesis of M cannot be carried out as in the case of M,, but the partial synthesis below will give fairly good results. The proposed formula is

11'=-p∙∞⅛,h^

⅜=K, <SS).

These formulæ have been used in the example of the computation of a tide-table given in the Admiralty Scientific Manual (1886).

§ 31. Synthesis of Lunar and Solar Semi-Diurnal Tides.

Let A be the excess of 〗’s over ⌾'s R. A., so that A = a-a,, )

≠, = ≠ + A, t (86).

and h2=M cos 2(ψ-μ) + M, cos 2(ψ + A-μ,) J

The synthesis is then completed by writing

H cos 2(μ - φ) = M + M, cos 2( A - μ, + μ), Hsin2(μ-ϕ)= M,sin2(A-μ,+μ),

so that hs=H cos 2(ψ-φ) (87).

Then H is the height of the total semi-diurnal tide and φ∕(γ - da∣dt) or φ∕(γ - σ) or 2/29 φ, when φ is given in degrees, is the “ interval ” from the moon’s transit to high water.

The formulæ for H and φ may be written

H = √ {M2+M,2 + 2MM, cos 2(A -μl +μ)} )

tan 2(u - ⅛) = M'sin 2(A ~ μ' ⅛)- l· (88).

ιanzlμ φ) M+M, coβ2(A-∕⅛+∕t) J

They may be reduced to a form adapted for logarithmic calculation. Since A goes through its period in a lunation, it follows that H and φ have inequalities with a period of half a lunation. These are called the “fortnightly or semi-menstrual inequalities” in the height and interval.

Spring tide obviously occurs when A=μ,- μ. Since the mean value of A is s - h (the difference of the mean longitudes), and since the mean values of μ and μ, are ½κm, ½κ8, it follows that the mean value of the period elapsing after full moon and change of moon up to spring tide is (κ8 - κm)∣2(σ - η). The association of spring tide with full and change is obvious, and a fiction has been adopted by which it is held that spring tide is generated in those configura­tions of the moon and sun, but takes some time to reach the port of observation. Accordingly (κ8 - κm)∕2(σ - η) has been called the “age of the tide.” The average age is about 36 hours as far as observations have yet been made. The age of the tide appears not in general to differ very much from the ages of the declinational and parallactic inequalities.

In computing a tide-table it is found practically convenient not to use A, which is the difference of R. A.’s at the unknown time of high water, but to refer the tide to A0, the difference of R.A.’s at the time of the moon’s transit. It is clear that A0 is the apparent time of the moon’s transit reduced to angle at 15o per hour. We have already remarked that φ∕(γ - da∣dt) is the interval from transit to high water, and hence at high water

A = a0+d <89)∙

As an approximation we may attribute to all the quantities in the second term their mean values, and we then have

σ-r

A=A0 + -A

u y-σ

and A.-μl+ μ≈A9-μlV^-f⅛=⅛>~lil + ⅜⅛lt (θθ)∙

This approximate formula (90) may be used in computing from (88) the fortnightly inequality in the “height” and “interval.”

In this investigation we have supposed that the declinational and parallactic corrections are applied to the lunar and solar tides be­fore their synthesis ; but it is obvious that the process might be reversed, and that we may form a table of the fortnightly inequality based on mean values Hm and H8, and afterwards apply corrections. This is the process usually adopted, but it is less exact. The labour of computing the fortnightly inequality, especially by graphical methods, is not great, and the plan here suggested seems preferable.

@@@1 The tide has been referred by Lubbock and others to an earlier transit, and not to the one immediately preceding the time under consideration. In this case we cannot admit with great accuracy that *da∣dt=σ,* since the interval may be 30 or 40 hours.