§ 32. Diurnal Tides.

These tides have not been usually treated with completeness in the synthetic method. In the tide-tables of the British Admiralty we find that the tides at some ports are “affected by diurnal in­equality ” ; such a statement may be interpreted as meaning that the tides are not to be predicted by the information given in the so-called tide-table. The diurnal tides are indeed complex, and do not lend themselves easily to a complete synthesis. In the har­monic notation the three important tides are K1,O,P, and the lunar portion of K1 is nearly equal to O in height, whilst the solar portion is nearly equal to P. A complete synthesis may be carried out on the lines adopted in treating the semi-diurnal tides, but the ad­vantage of the plan is lost in consequence of large oscillations of the amplitude through the value zero, so that the tide is often represented by a negative quantity multiplied by a circular function. It is best, then, only to attempt a partial synthesis, and to admit the existence of two diurnal tides.

We see from schedules [A, ii.] and [B, i.], § 23, that the principal diurnal tides are those lettered O, P, K1. Of these K1 occurs both for the moon and the sun. The synthesis of the two parts of K1 is effected without difficulty, and the result is a formula for the total K1 tide like that in [A, ii,], but with the v which occurs in the argu­ment replaced by a different angle denoted as v'. If, then, we write

V0=t+h-2s-v + 2ξ+½π (91.)

V=t + h-v'-½π ∕ (°1)>

the three tides O,K1,P are written as follows :—

0 =f0 Ho cos (V0-Kø), K1 = f H'cos(V'-√), P = - Hj, cos [V'- κ'- (2Ä - ∕) + κ'- xp)] (92).

The last two tides have very nearly the same speed, so that we may assume κ' = κp, and that Hp has the same ratio to H' as in the equilibrium theory. Now, in schedules [A, ii.], [B, ii.], § 23, the coefficient of K1, viz., H' (the sum of the lunar and solar parts), is •26522, and the coefficient of P, viz., Hp, is ⋅08775, so that H' = 3⋅023 Hp, or say=3Hp. Hence we have

K1 + P=H' [f' - ⅓ cos (2h - v')] cos (V' - κ)

- H' ⅓ sin(2h - v') sin (V' - κ').

If, therefore, we put

R'cosψ=H' [f' - ⅓ cos (2h - v')] Ί

R' sin ψ=⅓H' sin (2h-v') J W>

K1 + P=R'cos ( V'+ψ - κ').

It is clear that ψ and R' have a semi-annual inequality, and there­fore for several weeks together R' and ψ may be treated as constant.

Now suppose that we compute V0 and V' at the epoch—that is, at the initial noon of the period during which we wish to predict the tides—and with these values put

ζ0=κ0 - V0 at epoch, ζ'=κ' - V' at epoch - ψ.

Then the speed of V0 is γ-2σ, or 13o∙94303 per hour, or 360°-25°∙3673 per day; and the speed of V' is γ, or 15°⋅0410686 per hour, or 360o∙9856 per day. Hence, if t be the mean solar time on the (n+1)th day since the initial moment or epoch,

V0 - κ0=360on + 13o∙943 t - ζ0 - 25o∙367n,

V' + ψ-κ'=360on + 15o∙041 t-ζ'+ 0°∙986n.

Therefore the diurnal tides at time t of the (n + 1)th day are given by 0=f0H0cos[130∙943t-f0-250∙367n] 1 ,0.×

K1+P=R' cos [15o∙0411 - f + 0o∙986n] J '∙y4j'

If we substitute for t the time of high or low water as computed simply from the semi-diurnal tide, it is clear that the sum of these two expressions will give the diurnal correction for height of tide at high or low water, provided the diurnal tides are not very large. If we consider the maximum of a function

A cos 2(t - a) + B cos n(t - β),

where B is small compared with A and n is nearly unity, we see that the time of maximum is given approximately by t = a, with a correction δt determined from

-2A sin (2δt)-nB sin n(a-β) = 0 ;

180° 7iB . ox

or ot=——. smw(α-β).

In this way we find that the corrections to the time of high water from 0 and K1 + P are

δt0= - 0h∙988( 1 sin[13o∙943t- f0- 25β∙367n] )

∕ ∖ R' ? (θ∙\*),

δt'= -0h∙988( 1+—-) gsin[15o∙041t-f + 0°∙986n] )

H denoting the height and t the time of high water as computed from the semi-diurnal tide. If t next denotes the time of low water the same corrections with opposite sign give the corrections for low water.

If the diurnal tides are large a second approximation will be necessary. These formulæ have been used in computing a tide-table in the example given in the Admiralty Scientific Manual (1886).

§ 33. Explanation of Tidal Terms in common use ; Datum Levels.

The mean height at spring tide between high and low water is called the spring rise, and is equal to 2(Hm + Hs). The height between mean high-water mark of neap tide and mean low-water mark at spring tide is called the neap rise, and is equal to 2Hm. The mean height at neap tide between high and low water is called the neap range ; this is equal to 2(Hm - Hs). Neap range is usually about one-third of spring range. The mean period between full or change of moon and spring tide is called the age of the tide ; this is equal to (κs- κm)∕2(σ - η), or, if κs - κm be expressed in degrees, 0h-984 × (κs - κm) ; κ8 - κm is commonly about 36o, and the age about 36h. The period elapsing from the moon’s upper or lower transit until it is high water is called the interval or the lunitidal interval. The interval at full moon or change of moon is called the establish­ment of the port or the vulgar establishment. The interval at spring tide is called the corrected or mean establishment.

The mean establishment may be found from the vulgar establish­ment by means of the spring and neap rise and the age of the tide, as follows.

Let a be the age of the tide reduced to angle at the rate of 1o∙016 to the hour. Then the mean establishment in hours is equal to the vulgar establishment in hours, diminished by a period expressed in hours numerically equal to 1/29 of the angle whose tangent is Hs sin a∕(Hm+Hs cos a), expressed in degrees. Also Hs∕Hm is equal to the ratio of the excess of spring rise over neap rise to neap rise.

The French have called a quantity which appears to be identical with Hm + Hs, or half the spring rise, the unit of height, and then define the height of any other tide by a tidal coefficient.@@1

The practice of the British Admiralty is to refer their soundings and tide-tables to “mean low-water mark of ordinary spring tides.” This datum is found by taking the mean of the low-water marks of such observations at spring tide as are available, or, if the obser­vations are very extensive, by excluding from the mean such spring tides as appear to be abnormal, owing to the largeness of the moon’s parallax at the time or any other cause. The Admiralty datum is not, then, susceptible of exact scientific definition ; but, when it has once been fixed with reference to a bench mark ashore, it is expedient to adhere to it, by whatever process it was first fixed.

It is now proposed to adopt for any new Indian tidal stations a low-water datum for the tide-table to be called “ Indian low-water mark,”@@2 and to be defined as Hm + H8 + H' + H0 below mean-water level. Although such a datum is not chosen from any precise scientific considerations, it is susceptible of exact definition, is low enough to exclude almost all negative entries from the table (a sine qua non for a good datum), and will differ but little from the Admiralty datum, however that may be determined. A valuable list of datum levels is given by Mr J. Shoolbred in a Report to the British Association in 1879.

§ 34. On the Reduction of Observations of High and Low Water.@@3

A continuous register of the tide or observation at fixed intervals of time, such as each hour, is certainly the best ; but for the adequate use of such a record some plan analogous to harmonic analysis is necessary. Observations of high and low water only have, at least until recently, been more usual. Some care has to be taken with respect to these observations, for about high and low water an irregularity in the rise and fall becomes very noticeable, especially if the place of observation is badly chosen.@@4 Observa­tions should therefore be taken every five or ten minutes for half an hour or an hour, embracing the time of high and low water. The time and height of high and low water should then be found by plotting down a curve of heights, and by taking as the true tide-curve a line which presents a sweeping curvature and smoothes away the minor irregularities. A similar but less elaborate process would render hourly observations more perfect. In the reduction the immediate object is to connect the times and heights of high and low water with the moon’s transits by means of the establish­ment, age, and fortnightly inequality in the interval and height. The reference of the tide to the establishment is not, however, scientifically desirable, and it is better to determine the mean establishment, which is the mean interval from the moon’s transit to high water at spring tide, and the age of the tide, which is the mean period from full moon and change of moon to spring tide.

For these purposes the observations may be conveniently treated graphically.@@5 An equally divided horizontal scale is taken to represent the twelve hours of the clock of civil time, regulated to the time of the port, or—more accurately—arranged always to show

@@@1 See Hatt, Phénomène *des Marées,* p. 151, Paris, 1885.

@@@2 See Prefaces to *Indian official Tide-Tables for 1887.*

@@@3 Founded on Whewell’s article “ Tides ”, in *Admiralty Sc. Manual* (ed. 1841), and on Airy’s “Tides and Waves,” in *Ency. Metroρ.*

@@@4 Waves with a period of from five to twenty minutes are very common, and appear to be analogous to the “seiches” of Geneva and other lakes. See Forel, *Bulletin Soc. Vaud. Sci. Nat.,* 1873, 1875, 1877, and 1879; *Ann. Chimie et Physique,* vol. ix,, 1876 ; *Comptes Rendus,* 1879 ; *Arch. Sci., Ph., et Nat., Geneva,* 1885 ; also Airy, “ On the Tides of Malta,” *Phil. Trans.,* 1878, part i.

@@@5 For a numerical treatment, see *Directions for Reducing Tidal Observations,* by Commander Burdwood, R.N., London, 1876.