quently in estuaries the tank may be dispensed with. At any rate we suppose that water is provided rising and falling with the tide, without much wave-motion. The nature of the installation de­pends entirely on the circumstances of the case. A vertical pipe is fixed in the water in such a way as to admit it only through holes small enough to annul wave-motion and large enough to make no sensible retardation of its rise and fall in the pipe. The diameter of the pipe differs greatly in different instruments : some­times that which we have described as the tank serves as the pipe, and sometimes the pipe alone dips into the sea. A cylindrical float, usually a hollow metallic box or a block of green-heart wood, hangs and floats in the pipe, and is of such density as just to sink without support. In Sir W. Thomson’s gauge the float hangs by a fine platinum wire, in Newman’s (used in India) by a metallic ribbon. In the latter a chain hangs at the bottom of the float of such weight that, whether the water be high or low, there is the same upward force on the float. It is necessary that the pull on the float should be constant, otherwise a systematic error is intro­duced between rising and falling water. The suspension wire is wrapped round a wheel, and imparts to it rotation proportional to the rise and fall of tide. By a simple gearing this wheel drives another, by which the range is reduced to any convenient extent. A fine wire wound on the final wheel of the train drags a pencil or pen up and down or to and fro proportionately to the tidal oscillations. The pencil is lightly pressed against a drum, which is driven by clockwork so as to make one revolution per day. The pen leaves its trace or tide-curve on paper wrapped round the drum. Generally, however, the paper is fixed to the drum, and the record of a fortnight may be taken without change of paper. An example of a tide-curve for Apollo Bunder, Bombay, from 1st to 15th January 1884, is shown in fig. 3. Sometimes the paper is in a long band, which the drum picks off from one coil and delivers on to another. The contact of the pen must be such that the work done in dragging it over the paper is small, otherwise a varying tension is thrown on to the float wire. Hence, if the fric­tion is considerable, the float must be large.

The conditions necessary for a good tide-gauge appear to be better satisfied by Sir W. Thomson’s than by any other ; but, as his in­strument is recent, other forms have been much more extensively used, and have worked well. The peculiarity of Thomson’s tide­gauge is that, by giving the drum an inclination to the vertical, the pressure of the pen on the paper and on its guides is very deli­cately regulated to the minimum necessary for effecting the purpose. In other gauges the drum has been either vertical or horizontal, and the amount of friction has necessarily been considerably greater.@@1

§ 37. The Harmonic Analyser.

If a function H be expressed as a series of harmonic terms, and if one pair of these terms be A cos nt + B sin nt, then, if T be a multiple of the complete period 2π/n, we have

A=y,, ∕^1Hcos ntdt, B=⅜⅛ fτHwa,ntdt.

1J o Uo

Thus a machine which will effect these integrations will give A and B. Such a machine has been invented by Prof. James Thomson and perfected by Sir W. Thomson. In fig. 4 let TT' be a circular table, capable of rotation about the inclined shaft s. Let S be a sphere touching the table anywhere along its horizontal diameter. Let C be a cylinder, of somewhat smaller diam­eter than the sphere, capable of rota­tion about a horizontal axis parallel to the table, and touching the sphere so that CS is parallel to TT'. Sup­pose that the point of contact of the sphere with the table is distant x from the centre of the table, and

nearer to us than the shaft ; then, when the shaft s and the table TT' turn in such a direction that T rises from the paper and T' goes below it, the sphere will turn in the direction of its arrow. If the radius of the sphere is a, and that of the cylinder b, then, when the table turns through a small angle δθ, the sphere turns through xδθ∣a and the cylinder through xδθ∣b. This angle vanishes if S touches the table at the centre, and is reversed if the sphere be moved across to the other side of the centre. Also whilst the table is turning the sphere may be rolled backwards and forwards without rubbing, and thus transmits motion from the table to the cylinder without slipping. Now suppose the turning of the table is so constrained that δθ=kcos ψdψ, whilst x is constrained to be equal to the arbitrarily varying quantity H. Then the total angle turned through by the cylinder, as the machine runs, is proportional to∫Hcοsψdψ. If we impart to the table a simple

harmonic oscillatory motion, with a period proportional to the lunar half-day, whilst the sphere moves, relatively to the centre of the table, proportionately to the tide-heights on the same time-scale, then, at the end of a sufficient number of lunar days, we shall find that the total angle turned through by the cylinder is proportional to either the A or B component of the lunar semi-diurnal tide. An index, which points to a dial, may be fixed to the cylinder, so that the required result may be read off.

In the harmonic analyser the tide-curve diagram is wrapped on a drum, which is turned by one hand, whilst with the other a pointer is guided to follow the tide-curve. As the drum turns proportion­ately to mean solar time, appropriate gearing causes two tables to execute harmonic oscillations in phases at right angles, with lunar semi-diurnal period. At the same time a fork attached to the pointer guides the two spheres so that their distances from the centres of their tables are equal to the tide-height in the diagram. The in­dexes attached to the two cylinders give the two components of the lunar semi-diurnal tide, and the approximation improves the longer the tide-curve which is passed through the machine. Corresponding to each of the principal lunar and solar tides there are a pair of tables, spheres with guiding forks, and cylinders similarly geared, and there is another sphere and another table, which last always turns the same way and at the same rate as the drum, from which the mean height of water is determined. Such an instrument has been constructed under the supervision of Sir W. Thomson, but has not yet been put into practical use, so that we cannot say how it will compete with the arithmetical harmonic analysis. A similar, but less complex machine for the analysis of meteorological observations is in constant use in the Meteorological Office in London, and is found to work well.@@2

§ 38. The Tide-Predicting Instrument.

The first suggestion for instrumental prediction of tides was given, we believe, by Sir W. Thomson in 1872, and the instruments since made have been founded on the principles which he then laid down. Mr Edward Roberts bore a very important part in the first practical realization of such a machine, and a tide-predicter was constructed by Légé for the Indian Government under his direction. Thomson’s is the only instrument in Europe as yet in regular practical use for navigational purposes. It requires much skill and care in manipu­lation, and it has been ably worked by Mr Roberts for the produc­tion of the Indian tide-tables ever since its completion. We refer the reader to Sir W. Thomson’s paper on “Tidal Instruments,” in Inst. C.E., vol. lxv., and to the subsequent discussion, for a full account of the several instruments, and for details of the share borne by the various persons concerned in the realization of the idea.

Fig. 5 illustrates diagrammatically the nature of the instrument. A cord passes over and under a succession of pulleys, being fixed at one end and having at the other a pen which touches a revolving drum. If all the pulleys but one be fixed, and if that one executes a simple harmonic motion up and down, the pen will exe­cute the same motion with half amplitude. If a second pulley be now given an harmonic motion, the pen takes it up also with half amplitude. The same is true if all the pulleys are in harmonic motion. Thus the pen sums them all up, and leaves a trace on the revolving drum. When the drum and pul­leys are so geared that the angular motion of the drum is proportional to mean solar time, whilst the har­monic motions of the pulleys cor­respond in range and phase to all the important lunar and solar tides, the trace on the drum is a tide-curve, from which a tide-table may be constructed. The harmonic motion of the pulley is given by an arrangement in­dicated only in the case of the lower pulley in the figure. The pulley frame has attached to its vertical portion a horizontal slot, in which slides a pin fixed to a wheel. Suppose that whilst the drum turns through 15" the wheel turns through 28o⋅984. Now a lunar day is 24⋅842 mean solar hours ; hence as the drum turns through 15° × 24⋅842 the wheel turns through 24⋅842 × 28o⋅984 or 720o. Thus, if the drum turns with an angular velocity pro­portional to solar time, the wheel turns with twice the angular velocity proportional to lunar time, and the pulley geared to the wheel executes lunar semi-diurnal harmonic oscillations. When the throw of the pin and its angular position on its wheel are adjusted so as to correspond with the range and phase of the observed lunar semi­diurnal tide, the oscillation of the pulley remains rigorously ac­curate for that tide for all future time, if the gearing be rigorously accurate, and with all needful accuracy for some ten years of tide

@@@1 For further details concerning the establishment of tide-gauges, see Major Braid’s *Manual of Tidal Observation,* London, 1887, and Sir W. Thomson, “On Tidal Instrumente,” in *Inst. Civ. Eng.,* vol. lxv. p. 10.

@@@2 For further details, see Appendixes iii., iv., v., to Thomson and Tait’s *Nat. Phil.,* 1879, vol. i., part i.; James Thomson, *Proc. Roy. Soc.,* vol. xxiv., 1876, p. 262, and (Sir W. Thomson) pp. 269, 271 ; Sir W. Thomson, *Proc. Inst. C.E.,* vol. lxv.