tardation ; and this is the case so long as the viscosity is not great. The rigorous result for a viscous planet shows that in general the obliquity will increase, and it appears that, with small viscosity of the planet, if the period of the satellite be longer than two periods of rotation of the planet, the obliquity increases, and vice versa. Hence zero obliquity is only dynamically stable when the period of the satellite is less than two periods of the planet’s rotation.

Suppose the motions of the planet and of its solitary satellite to be referred to the invariable plane of the system. The axis of resultant moment of momentum is normal to this plane, and the component rotations are that of the planet about its axis of figure and the orbital motion of the planet and satellite round their com- mon centre of inertia ; the axis of this latter rotation is clearly the normal to the satellite’s orbit. Hence the normal to the orbit, the axis of resultant moment of momentum, and the planet’s axis of rotation must always lie in one plane. From this it follows that the orbit and the planet’s equator must necessarily have a common node on the invariable plane. If either of the component rotations alters in amount or direction, a corresponding change must take place in the other, such as will keep the resultant moment of momentum constant in direction and magnitude. It has been shown that the effect of tidal friction is to increase the distance of the satellite from the planet, and to transfer moment of momentum from that of planetary rotation to that of orbital motion. If, then, the direction of the planet’s axis of rotation does not change, it follows that the normal to the lunar orbit must approach the axis of resultant moment of momentum. By drawing a series of parallelograms on the same diameter and keeping one side constant in direction, this may be easily seen to be true. This is equivalent to saying that the inclination of the satellite’s orbit will decrease. But this decrease of inclination does not always necessarily take place, for the previous investigations show that another effect of tidal friction may be to increase the obliquity of the planet’s equator to the invariable plane, or, in other words, to increase the inclination of the planet’s axis to the axis of resultant moment of momentum. Now, if a parallelogram be drawn with a constant diameter, it is seen that by increasing the inclination of one of the sides to the diameter (and even decreasing its length) the inclination of the other side to the diameter may also be in­creased. The most favourable case for such a change is when the side whose inclination is increased is nearly as long as the diameter. From this it follows that the inclination of the satellite’s orbit to the invariable plane may increase, and that it is most likely to increase, when the moment of momentum of planetary rotation is large com­pared with that of the orbital motion. The analytical solution of the problem agrees with these results, for it shows that if the vis­cosity of the planet be small the inclination of the orbit always diminishes, but if the viscosity be large, and if the satellite moves with a short periodic time (as estimated in rotations of the planet), the inclination of the orbit will increase. These results convey some idea of the physical causes which may have given rise to the present inclination of the lunar orbit to the ecliptic. For the analytical investigation shows that the inclination of the lunar orbit to a certain plane, which replaces the invariable plane when the solar attraction is introduced, was initially small, that it then increased to a maximum, and that it finally diminished and is still diminishing.

But the laws above referred to would, by themselves, afford a very unsatisfactory explanation of the inclination of the lunar orbit, be­cause the sun’s attraction is a matter of much importance. It has been found that, if the viscosity of the planet be small, the in­clination of the orbit of the solitary satellite to the invariable plane will always diminish ; but, when solar influence is introduced, the corresponding statement is not true with regard to the inclination of the lunar orbit to the proper plane, for during one part of the moon’s history the inclination to the proper plane would have increased even if the viscosity of the earth had been small.

Consider a satellite revolving about a planet in an elliptic orbit, with a periodic time which is long compared with the period of rota­tion of the planet ; and suppose that frictional tides are raised on the planet. The major axis of the tidal spheroid always points in advance of the satellite, and exercises on it a force which tends to accelerate its linear velocity. When the satellite is in perigee the tides are higher, and this disturbing force is greater than when the satellite is in apogee. The disturbing force may therefore be repre­sented as a constant force, always tending to accelerate the motion of the satellite, and as a periodic force which accelerates in perigee and retards in apogee. The constant force causes a secular increase of the satellite’s mean distance and a retardation of its mean motion. The accelerating force in perigee causes the satellite to swing out further than it would otherwise have done, so that when it comes round to apogee it is more remote from the planet. The retarding force in apogee acts exactly inversely, and diminishes the perigean distance. Thus, the apogean distance increases and the perigean distance diminishes, or, in other words, the eccentricity of the orbit increases. Now consider another case, and suppose the satellite’s periodic time to be identical with that of the planet’s rotation. Then, when the satellite is in perigee, it is moving faster than the planet rotates, and when in apogee it is moving slower ; hence at apogee the tides lag, and at perigee they are accelerated. Now the lagging apogean tides give rise to an accelerating force on the satellite, and increase the perigean distance, whilst the accelerated perigean tides give rise to a retarding force, and decrease the apogean distance. Hence in this case the eccentricity of the orbit will diminish. It follows from these two results that there must be some intermediate periodic time of the satellite for which the eccentricity does not tend to vary.

But the preceding general explanation is in reality somewhat less satisfactory than it seems, because it does not make clear the existence of certain antagonistic influences, to which, however, we shall not refer. The rigorous result, for a viscous planet, shows that in general the eccentricity of the orbit will increase ; but, if the obliquity of the planet’s equator be nearly 90o, or if the viscosity be so great as to approach perfect rigidity, or if the periodic time of the satellite (measured in rotations of the planet) be short, the eccentricity will slowly diminish. When the viscosity is small the law of variation of eccentricity is very simple : if eleven periods of the satellite occupy a longer time than eighteen rotations of the planet, the eccentricity increases, and vice versa. Hence in the case of small viscosity a circular orbit is only dynamically stable if the eleven periods are shorter than the eighteen rotations.

X. Cosmogonic Speculations founded on Tidal Friction.

§ 50. History of the Earth and Moon.

We shall not attempt to discuss the mathematical methods by which the complete history of a planet, attended by one or more satellites, is to be traced. The laws indicated in the preceding sections show that there is such a problem, and that it may be solved, and we refer to Mr Darwin’s papers for details (Phil. Trans., 1879-81). It may be interesting, however, to give the various results of the investigation in the form of a sketch of the possible evolution of the earth and moon, followed by remarks on the other planetary systems and on the solar system as a whole.

We begin with a planet not very much more than 8000 miles in diameter, and probably partly solid, partly fluid, and partly gaseous. It is rotating about an axis inclined at about 11° or 12o to the nor­mal to the ecliptic, with a period of from two to four hours, and is revolving about the sun with a period not much shorter than our present year. The rapidity of the planet’s rotation causes so great a compression of its figure that it cannot continue to exist in an ellipsoidal form with stability ; or else it is so nearly unstable that complete instability is induced by the solar tides. The planet then separates into two masses, the larger being the earth and the smaller the moon. It is not attempted to define the mode of separation, or to say whether the moon was initially a chain of meteorites. At any rate it must be assumed that the smaller mass became more or less conglomerated and finally fused into a spheroid, perhaps in consequence of impacts between its constituent mete­orites, which were once part of the primeval planet. Up to this point the history is largely speculative, for the conditions of insta­bility of a rotating mass of fluid have not yet been fully investigated. We now have the earth and moon nearly in contact with one another, and rotating nearly as though they were parts of one rigid body.@@1 This is the system which was the subject of dynamical investigation. As the two masses are not rigid, the attraction of each distorts the other ; and, if they do not move rigorously with the same periodic time, each raises a tide in the other. Also the sun raises tides in both. In consequence of the frictional resistance to these tidal motions, such a system is dynamically unstable. If the moon had moved orbitally a little faster than the earth rotated, she must have fallen back into the earth ; thus the existence of the moon compels us to believe that the equilibrium broke down by the moon revolving orbitally a little slower than the earth rotates. In consequence of the tidal friction the periodic times both of the moon (or the month) and of the earth’s rotation (or the day) increase ; but the month increases in length at a much greater rate than the day. At some early stage in the history of the system the moon was conglomerated into a spheroidal form, and acquired a rotation about an axis nearly parallel to that of the earth.

The axial rotation of the moon is retarded by the attraction of the earth on the tides raised in the moon, and this retardation takes place at a far greater rate than the similar retardation of the earth’s rotation. As soon as the moon rotates round her axis with twice the angular velocity with which she revolves in her orbit, the position of her axis of rotation (parallel with the earth’s axis) becomes dynamically unstable. The obliquity of the lunar equator to the plane of the orbit increases, attains a maximum, and then diminishes. Meanwhile the lunar axial rotation is being reduced towards identity with the orbital motion. Finally, her equator is nearly coincident with the plane of the orbit, and the attraction of the earth on a tide, which degenerates into a permanent ellipticity

@@@1 See criticisms by Mr Nolan, *Genesis of Moon,* Melbourne, 1885 ; also *Nature,* 18th February 1886.