was far beyond what they seem to have considered neces­sary, as they only record astronomical phenomena *(e.g.,* eclipses, occultations) as having occurred “towards the middle of the third hour,” or “ about 8⅓ hours of the night,” without ever giving minutes.@@1 The Arabians had a clearer perception of the importance of knowing the accurate time of phenomena, and in the year 829 we find it stated that at the commencement of the solar eclipse on 30th November the altitude of the sun was 7° and at the end 24°, as observed at Baghdad by Ahmed ibn Abdallah, called Habash.@@2 This seems to be the earliest determina­tion of time by an altitude ; and this method then came into general use among the Arabians, who on observing lunar eclipses never failed to measure the altitude of some bright star at the beginning and end of the eclipse. In Europe this method was adopted by Purbach and Regio­montanus, apparently for the first time in 1457. Bernhard Walther, a pupil of the latter, seems to have been the first to use for scientific purposes clocks driven by weights : he states that on 16th January 1484 he observed the rising of the planet Mercury and immediately attached the weight to a clock having an hour-wheel with fifty-six teeth; at sunrise one hour and thirty-five teeth had passed, so that the interval was an hour and thirty-seven minutes. For nearly two hundred years, until the application of the pendulum to clocks became general, astronomers could place little or no reliance on their clocks, and consequently it was always necessary to fix the moment of an ob­servation by a simultaneous time determination. For this purpose Tycho Brahe employed altitudes observed with quadrants ; but he remarks that they are not always of value, for if the star is taken too near the meridian the altitude varies too slowly, and if too near the horizon the refraction (which at that time was very imperfectly known) introduces an element of uncertainty. He therefore pre­ferred azimuths, or with the large “armillary spheres” which played so important a part among his instruments he measured hour-angles or distances from the meridian along the equator.@@3 Transits of stars across the meridian were also observed with the meridian quadrant, an instru­ment which is alluded to by Ptolemy and was certainly in use at the Marágha (Persia) observatory in the 13th cen­tury, but of which Tycho was the first to make extensive use. It appears, however, that he chiefly employed it for determining star-places, having obtained the clock error by the methods already described.

In addition to these methods, that of “ equal altitudes ” was much in use during the 17th century. That equal distances east and west of the meridian correspond to equal altitudes had of course been known as long as sun-dials had been used ; but, now that quadrants, cross-staves, and parallactic rules @@4 were commonly employed for measuring altitudes more accurately, the idea naturally suggested it­self to determine the time of a star’s or the sun’s meridian passage by noting the moments when it reached any par­ticular altitude on both sides of the meridian. But Tycho’s plan of an instrument fixed in the meridian was not for­gotten, and from the end of the 17th century, when Roemer invented the transit instrument, the observation of transits across the meridian became the principal means of deter­mining time at fixed observatories, while the observation of altitudes, first by portable quadrants, afterwards by re­flecting sextants, and during the 19th century by port­able alt-azimuths or theodolites, has been used on journeys.

During the last fifty years the small transit instrument, with what is known as a “ broken telescope,” has also been much employed on scientific expeditions ; but great caution is necessary in using it, as the difficulties of getting a per­fectly rigid mounting for the prism or mirror which reflects the rays from the object-glass through the axis to the eye­piece appear to be very great, for strange discrepancies in the results have often been noticed. The gradual develop­ment of astronomical instruments has been accompanied by a corresponding development in timekeepers. From being very untrustworthy, astronomical clocks are now made to great perfection by the application of the pendu­lum and by its compensation, while the invention of chronometers has placed a portable and equally trust­worthy timekeeper in the hands of travellers.

We shall now give a sketch of the principal methods of determining time.

In the spherical triangle ZPS between the zenith, the pole, and a star the side ZP=90°-ϕ (ϕ being the latitude), BS=90o -δ (δ being the declination), and ZS or Z=90° minus the observed alti­tude. The angle ZPS=t is the star’s hour-angle or, in time, the interval between the moment of observation and the meridian pass­age of the star. We have then

. cos Z— sin φ sin δ cos t = — ,

cos φ cos δ

which formula can be made more convenient for the use of logarithms by putting Z+φ + δ=2S, which gives

. 51. sin(<S,-≠)sin(N-δ)

tam it— ——.

∙4 cos ό cos(0 - Z)

According as the star was observed west or east of the meridian, t will be positive or negative. If a be the right ascension of the star, the sidereal time =t + a, a as well as δ being taken from an ephemeris. If the sun had been observed, the hour-angle t would be the apparent solar time. The altitude observed must be cor­rected for refraction, and in the case of the sun also for parallax, while the sun’s semi-diameter must be added or subtracted, accord­ing as the lower or upper limb was observed. The declination of the sun being variable, and being given in the ephemerides for noon of each day, allowance must be made for this by interpolating with an approximate value of the time. As the altitude changes very slowly near the meridian, this method is most advantageous if the star be taken near the prime vertical, while it is also easy to see that the greater the latitude the more uncertain the result. If a number of altitudes of the same object are observed, it is not necessary to deduce the clock error separately from each observa­tion, but a correction may be applied to the mean of the zenith distances. Supposing n observations to be taken at the moments T1, T2, T3,..., the mean of all being T0, and calling the z corre­sponding to this Z, we have

\*∙Λ⅞Ι-⅛4S⅞-V! ⅛=Z+f(71-7<,) + 15(¾-Γ0)>i and so on, t being the hour-angle answering to T0. As Σ( Τ - T0) =0, these equations give

¾÷⅞+⅞+... 1 d2Z(T1-T0)2 + (7τ2-T0)2+....

n 2 dt2 n

¾+⅝4\*i⅛4\* » ». d-Z 22 sin- ⅛( T — Tj)

~ γι di2 n

But, if in the above-mentioned triangle we designate the angles at Z and S by 180o - A and p, we have

sin z sin A =cos δ sin t ;

sin z cos A = — cos φ sin δ + sin φ cos δ cos t ;

and by differentiation

d2Z\_ cos φ cos δ cos A cosp dfi~ sinZ ’

in which A and p are determined by

sin ί . 1 . sin ί

sin A ≈ -.—cos δ and sin p = ——⅛ cos φ.

sin Z sin Z

With this corrected mean of the observed zenith distances the hour- angle and time are determined, and by comparison with Τ0 the error of the timekeeper.

The method of equal altitudes gives very simply the clock error equal to the right ascension minus half the sum of the clock times corresponding to the observed equal altitudes on both sides of the meridian. When the sun is observed, a correction has to be applied for the change of declination in the interval between the observa­tions. Calling this interval 2t, the correction to the apparent noon

@@@1 For astronomical purposes the ancients made use of mean-time hours—ώρα*ι* *ίσημϵριvaι, horæ equinoetiales—*into which they translated all indications expressed in civil hours of varying length—ῶρ*aι κaιρικaί, horæ temporales.* Ptolemy counts the mean day from noon.

@@@2 Caussin, Le livre de la grande table Hakémite, Paris, 1804, p. 100.

@@@3 See his Epistolæ astronomicæ, p. 73.

@@@4 See Navigation, vol. xvii. pp. 251 and 253.