venet’s *Spherical and Practical Astronomy,* vol. i. pp. 518- 542 and 550-557.

We now proceed to consider the four methods for find­ing the longitudes of fixed observatories, viz., by (1) moon culminations, (2) rockets or other signals, (3) transport of chronometers, and (4) transmission of time by the electric telegraph.

1. Moon Culminations.—Owing to the rapid orbital motion of the moon the sidereal time of its culmination is different for different meridians. If, therefore, the rate of the moon’s change of right ascension is known, it is easy from the observed time of culmina­tion at two stations to deduce their difference of longitude. Let the right ascension of the moon α and its differential coefficients be computed for the Greenwich time T, and let the culmination be observed at two places whose longitudes from Greenwich are λ and λ', the time of observation being T+t and T+t' Greenwich time, or in local time T+t+λ=θ and T+t' + λ'=θ' ; we have then

β∙-i=(1∙-i)g+∣(i∙.-i.)g+...,

and, as the difference of longitude is λ' -λ=(θ' - θ)- (t' -t), we have only to determine t’ -t from the first equation. This is simply done by a suitable selection of T. Calling T+½(t + t')= T', we have to put T'-½(t'-t) and T' + ½(t'-t) for T+t and T+tl. It is then easy to see that

\*'-\*=('-φ⅛<'-<)∙‰α.

and, solving this equation by first neglecting the second term on the right side aud then substituting the value of t' - t, thus found . ., 4,4, „ t θ,-θ iΓΘ'-θ-]3(Ρa

in that term, c t~da∣dt 24[jiα∕dJ dt3'

In order to be as much as possible independent of instrumental errors, some standard stars nearly on the parallel of the moon are observed at the two stations ; these “ moon-culminating stars ” are given in the ephemerides in order to secure that both observers take the same stars. As either the preceding or the following limb, not the centre, of the moon is observed, allowance must be made for the time the semi-diameter takes to pass the meridian and for the change of right ascension during this time. This method was proposed by Pigott towards the end of the 18th century, and has been much used ; but, though it may be very serviceable on journeys and expeditions to distant places where the chronometric and tele­graphic methods cannot be employed, it is not accurate enough for fixed observatories. This is due, not only to the difficulties attend­ing the observation (the difference of personal error in observing the moon and stars, the different apparent enlargement of the moon by irradiation in different telescopes and under different atmo­spheric circumstances, &c.), but chiefly to the large coefficient with which θ' - θ has to be multiplied in the final equation for λ' - λ. Errors of four to six seconds of time have therefore frequently been noticed in longitudes obtained by this method from a limited number of observations : the longitude of the Madras observatory was for many years assumed to be 5h 21m 3s⋅77, but subsequently by a telegraphic determination this was found to be 4s⋅37 too great.

2. Signals.—In 1671 Picard determined the difference of longi­tude between Copenhagen and the site of Tycho Brahe’s observa­tory by watching from the latter the covering and uncovering of a fire lighted on the top of the observatory tower at Copenhagen. Powder or rocket signals have been in use since the middle of the 18th century ; they are nowadays never used for this purpose, although several of the principal observatories of Europe were con­nected in this manner early in the 19th century.@@@1

3. Transport of Chronometers.—This means of determining longi­tude was first tried in cases where the chronometers could be brought the whole way by sea, but the improved means of communication on land led to its adoption in 1828 between the observatories at Greenwich and Cambridge, and in the following years between many other observatories. A few of the more extensive expedi­tions undertaken for this object deserve to be mentioned. In 1843 more than sixty chronometers were sent sixteen times backwards and forwards between Altona and Pulkowa, and in 1844 forty chronometers were sent the same number of times between Altona and Greenwich.@@@2 In 1844 the longitude of Valentia on the south­west coast of Ireland was determined by transporting thirty pocket chronometers via Liverpool and Kingstown and having an inter­mediate station at the latter place. The longitude of the United States naval observatory has been frequently determined from Greenwich. The following results will give an idea of the accuracy of the method.@@3

Previous to 1849, 373 chronometers 5h 8m 12s· 52

Expedition of 1849, Bond’s discussion 11s· 20

,, „ Walker’s ,, 12s·06

,, ,, Bond’s second result 12s∙26±0s∙20

,, 1855, 52 chronometers, 6 trips, Bond .. 13s·49±0s·19

The value now accepted from the telegraphic determination is 5h 8m 12s⋅09. The probable errors of the results for Pulkowa- Altona and Altona-Greenwich were supposed to be ±0s⋅039 and ±0s⋅042. It is of course only natural that the uncertainty of the results for the trans-Atlantic longitude should be much greater, considering the length of time which elapsed between the rating of the chronometers at the observatories of Boston (Cambridge, Massachusetts) and Liverpool. The difficulty of the method con­sists in determining the “travelling rate.” Each time a chrono­meter leaves the station A and returns to it the error is determined, and consequently the rate for the time occupied by the journeys from A to B and from B to A and by the sojourn at B. Similarly a rate is found by each departure from and return to B, and the time of rest at A and B is also utilized for determining the station­ary rate. In this way a series of rates for overlapping intervals of time are found, from which the travelling rates may be interpolated. It is owing to the uncertainty which necessarily attaches to the rate of a chronometer during long journeys, especially by land, where they are exposed to shaking and more or less violent motion, that it is desirable to employ a great number. It is scarcely neces­sary to mention that the temperature correction for each chrono­meter must be carefully investigated, and the local time rigorously determined at each station during the entire period of the operations.

4. Telegraphic Determination of Longitude.—This was first sug­gested by the American astronomer S. C. Walker, and owed its de­velopment to the United States Coast Survey, where it was employed from about 1849. Nearly all the more important public observa­tories on the continent of Europe have now been connected in this way, chiefly at the instigation of the “Europäische Gradmessung,” while the determinations in connexion with the transits of Venus and those carried out in recent years by the American and French Governments have completed the circuit of the greater part of the globe. The telegraphic method compares the local time at one station with that at the other by means of electric signals. If a signal is sent from the eastern station A at the local time T, and received at the western station B at the local time T1, then, if the time taken by the current to pass through the wire is called x, the difference of longitude is

λ=*T*-*T*1+*x*

and similarly, if a signal is sent from B at the time T2 and received at A at T3, we have λ=T3 - T2-x,

from which the unknown quantity x can be eliminated.

The operations of a telegraphic longitude determination can be arranged in two ways. Either the local time is determined at both stations and the clocks are compared by telegraph, or the time determinations are marked simultaneously on the two chronographs at the two stations, so that further signals for clock comparison are unnecessary. The first method has to be used when the tele­graph is only for a limited time each night at the disposal of the observers, or when the climatic conditions at the two stations are so different that clear weather cannot often be expected to occur at both simultaneously, also when the difference of longitude is so considerable that too much time would be lost at the eastern station waiting for the arrival of the transit record of one star from the western station before observing another star. The independent time determination also offers the advantage that the observations may be taken either by eye and ear or by the chronograph, and that the signals may be either audible beats of a relay or chrono­graphic signals, the rule being to have observations and signals made by similar operations. The best way of using audible beats of a relay is to let the circuit pass through an auxiliary clock, which from second to second alternately makes and breaks the current, the making of the current being rendered audible by the tapping of the relays at both stations. If, now, the auxiliary and the observing clocks are regulated to a different rate, the coinci­dences of the beats of the relay with those of the observing clock can be noted with great accuracy, from which the difference between the two observing clocks is found. It has been proved by experience that the degree of accuracy with which the clock comparison can be made by one coincidence is exactly equal to that of one chrono­graph signal, the probable error being in both cases about ±0s⋅015. It should, however, be mentioned that the interval between two consecutive coincidences cannot be made less than two minutes, whereas the chronograph signals may be given every second, and, as the observations made with the chronograph are also somewhat more accurate than those made by eye and ear, the chronograph should be used wherever possible. The other method, that of simultaneous registration at both stations of transits of the same stars, has also its advantages. Each transit observed at both stations furnishes a value of the difference of longitude, so that the final result is less dependent on the clock rate than in the first method, which necessitates the combination of a series of clock

@@@1 For instance, Greenwich and Paris in 1825 *(Phil. Trans.,* 1826). The result, 6m21s∙6, is only about 0s∙6 too great.

@@@2 As a great many of the chronometers used in 1844 were made by Dent and were of superior excellence, a smaller number was considered sufficient.

@@@3 Gould, *Transatlantic Longitude,* p. 5, Washington, 1869.