Germans and also numerous Greeks, English, and French. The population includes 26,000 Slavs, most of whom live in the country districts and are engaged in agriculture.

Trieste has been a free imperial port since 1719. It may be said to nearly monopolize the trade of the Adriatic, and has long eclipsed its ancient rival Venice. The annual value of its exports and imports is about 30 millions sterling. Among the chief im­ports are coffee, wine, fruit, grain, tobacco, petroleum, cotton, coals, and manufactured goods of various kinds ; the exports include spirits, liqueurs, sugar, meal, timber, glass, and machinery. Large quantities of fish are sent to Vienna. In 1885 the port was entered by 6971 vessels with an aggregate burden of 1,267,946 tons. The trading fleet of Trieste numbers about 500 ships of 100,000 tons burden. The chief shipping company is the Austrian Lloyd’s, founded in 1836, the steamers of which ply to the Mediterranean ports, Alexandria, Constantinople, the Black Sea, &c. The exten­sive wharfs and dockyards of the company lie to the south of the town. The chief branches of industry practised at Trieste are shipbuilding, soap-boiling, machine-making (especially marine engines), tanning, brewing, rope-making, and the manufacture of liqueurs (rosoglio). Trieste is the seat of government for the so- called Küstenland or Coast district, and is the seat of naval and military commanders and other officials. The town council, pre­sided over by the podestà, is also the diet of the crownland of Trieste (35 square miles). Trieste is the seat of the bishop of Capo d’Istria.

*History.—*At the time of the foundation of Aquileia by the Romans, the district which now includes Trieste was occupied by Celtic and Illyrian tribes ; and the Roman colony of Tergeste does not seem to have been established till the reign of Vespasian. After the break-up of the Roman dominion Trieste shared the general for­tunes of Istria and passed through various hands. From the em­peror Lothaire it received an independent existence under its count­bishops, and it maintained this position down to its capture by Venice in 1203. For the next 180 years its history consists chiefly of a series of conflicts with this city, which were finally put an end to by Trieste placing itself in 1382 under the protection of Leopold III. of Austria. The overlordship thus established in­sensibly developed into actual possession ; and except in the Napoleonic period (1797-1805 and 1809-1813) Trieste has since remained an integral part of the Austrian dominions,

TRIGGER-FISH. See File-Fish.

TRIGONOMETRY

TRIGONOMETRY is primarily the science which is concerned with the measurement of plane and spherical triangles, that is, with the determination of three of the parts of such triangles when the numerical values of the other three parts are given. Since any plane tri­angle can be divided into right-angled triangles, the solu­tion of all plane triangles can be reduced to that of right- angled triangles ; moreover, according to the theory of similar triangles, the ratios between pairs of sides of a right-angled triangle depend only upon the magnitude of the acute angles of the triangle, and may therefore be regarded as functions of either of these angles. The primary object of trigonometry, therefore, requires a classi­fication and numerical tabulation of these functions of an angular magnitude ; the science is, however, now under­stood to include the complete investigation not only of such of the properties of these functions as are necessary for the theoretical and practical solution of triangles but also of all their analytical properties. It appears that the solution of spherical triangles is effected by means of the same functions as are required in the case of plane triangles. The trigonometrical functions are employed in many branches of mathematical and physical science not directly concerned with the measurement of angles, and hence arises the importance of analytical trigonometry. The solution of triangles of which the sides are geodesic lines on a spheroidal surface requires the introduction of other functions than those required for the solution of triangles on a plane or spherical surface, and therefore gives rise to a new branch of science, which is from analogy frequently called spheroidal trigonometry. Every new class of surfaces which may be considered would have in this extended sense a trigonometry of its own, which would consist in an investigation of the nature and properties of the functions necessary for the measurement of the sides and angles of triangles bounded by geodesics drawn on such surfaces.

History.

An account of Greek trigonometry is given under Ptolemy *(q.v.).*

The Indians, who were much more apt calculators than the Greeks, availed themselves of the Greek geometry which came from Alexandria, and made it the basis of trigonometrical calculations. The principal improvement which they introduced consists in the formation of tables of half-chords or sines instead of chords. Like the Greeks, they divided the circumference of the circle into 360 degrees or 21,600 minutes, and they found the length in minutes of the arc which can be straightened out into the radius to be 3438'. The value of the ratio of the circumference of the circle to the diameter used to make this determination is 62832 :20000, or π = 3∙1416, which value was given by the astronomer Âryabhata (476-550; see Sanskrit, vol. xxi. p. 294) in a work called *Ârya- bhatîya,* written in verse, which was republished@@1 in Sanskrit by Dr Kern at Leyden in 1874. The relations between the sines and cosines of the same and of complementary arcs were known, and the formula sin ½*a* = √1719(3438 - cos a) was applied to the determination of the sine of a half angle when the sine and cosine of the whole angle were known. In the *Sûrya-Siddhânta,* an astronomical treatise which has been translated by Ebenezer Bourgess in vol. vi. of the *Journal* of the American Oriental Society (New Haven, 1860), the sines of angles at an interval of 3° 45' up to 90° are given ; these were probably obtained from the sines of 60° and 45° by continual application of the dimidi- ary formula given above and by the use of the complement­ary angle. The values sin 15o = 890', sin 7° 30' = 449', sin 3° 45' = 225', were thus obtained. Now the angle 3° 45' is itself 225' ; thus the arc and the sine of 1/96th of the cir­cumference were found to be the same, and consequently special importance was attached to this arc, which was called the right sine. From the tables of sines of angles at intervals of 3° 45' the law expressed by the equation sin *(n* +1.225') - sin (*n*. 225') = sin (*n*. 225') - sin (*n* - 1.225')

-sin[(*n*. 225')/(225)]

was discovered empirically, and used for the purpose of recalculation. Bhâskara (*fl*. 1150) used the method, to which we have now returned, of expressing sines and cosines as fractions of the radius ; he obtained the more correct values sin 3° 45'= 100/1529, cos 3° 45' = 466/467, and showed how to form a table, according to degrees, from the values sin 1° = 10/573, cos 1° = 6568/6569, which are much more accurate than Ptolemy’s values. The Indians did not apply their trigonometrical knowledge to the solution of triangles ; for astronomical purposes they solved right- angled plane and spherical triangles by geometry.

The Arabs were acquainted with Ptolemy’s *Almagest,* and they probably learned from the Indians the use of the sine. The celebrated astronomer of Batnæ, Abú 'Abdallah Mohammed b. Jābir al-Battání (Bategnius), who died in 929/930 A.D., and whose *Tables* were translated in the 12th century by Plato of Tivoli into Latin, under the title *De scientia stellarum,* employed the sine regularly, and was fully conscious of the advantage of the sine over the chord ; indeed, he remarks that the continual doubling is saved

@@@1 See also vol. ii. of the *Asiatic Researches* (Calcutta).