the displacement in tons and the number of tons required to increase the mean draught by 1 in., respectively, as ordinates (horizontal). The ordinate of the curve of displacement at any water line is clearly proportional to the area of the curve of tons per inch up to that water line.

The properties of the metacentric stability at small angles are used when determining the vertical position of the centre of gravity of a ship by an “ inclining experiment this gives a check on the calculations for this position made in the initial stages of the design, and enables the stability of the completed ship in any condition to be ascertained with great accuracy.

The experiment is made in the following manner

Let fig. 6 represent the transverse section of a ship; let *w, w* be two weights on deck at the positions P, Q, chosen as far apart trans- versely as convenient; and let G be the combined centre of gravity of ship and weights. When the weight at P is moved across the deck to Q', the centre of gravity of the whole moves from G to some point G' so that GG' is parallel to PQ' (assumed horizontal) and equal to *hw∣∖N* where *h* is the dis­tance moved through by P, and W is the total displacement. The ship in consequence heels to a small angle *θ,* the new vertical through G pass­ing through the meta­centre M; also GM = GG' cot *θ≈hw∕∖V* cot ø, the metacentric height being thereby determined and the position of G then found from the meta­centric diagram. In prac- tice *θ* is observed by means of plumb bobs or a short period pen­dulum recording angles on a cylinder;@@l the weight *w* at P, which is chosen so as to give a heel of from 3° to 5° is divided into several portions moved separately to Q'. The weight at *Q'* is replaced at P, the angle heeled through again observed; and the weight at Q similarly moved to P' where P'Q=*h*≡PQ', and the angle observed; GM is then taken as the mean of the various evaluations.

In the case of small transverse inclinations it has been assumed that the vertical through the upright and the inclined positions of the centre of buoyancy intersect, or, which is the same thing, that the centre of buoyancy remains in the same trans­verse plane when the vessel is inclined. This assumption is not generally correct for large transverse inclinations, but is nevertheless usually made in practice, being sufficiently accurate for the purpose of estimating the righting moments and ranges of stability of different ships, calculated under the same conventional system; this is all that is necessary for practical purposes.

With this assumption, there will always be a point of inter­section (M' in fig. 7) of the verticals through the upright and in- clined centres of buoyancy; and the righting lever is, as before, GZ = GM' sin *θ.* In this case, however, there is no simple formula for BM' as there is for BM in the limiting case where *θ* is infini­tesimal ; and other methods of calculation are necessary.

The development of this part of the subject was due originally to Atwood, who in the *Philosophical Transactions* of 1796 and 1798, advanced reasons for differing from the metacentric method which was published by Bouguer in his *Traite du navire* in 1746. Atwood’s treatment of stability (which was the foundation of the modes of calculation adopted in England until about twenty years ago) was as follows :—

Let λλ L, W'L' (fig. 7) be respectively the water lines of a ship when upright and inclined at an angle *θ,* S their point of intersection; B and B' the centres of buoyancy, *gι* and g2 the centres of gravity of the equal wedges WSW', L'SL, and Äi, *h2* the feet of the perpendiculars from gι, *gi* on the inclined water line. Draw GZ, BR parallel to W'L', meeting the vertical through B' in Z and R.

@@@1 Such an instrument is described by Froude for recording the “ relative ” inclination of a ship amongst waves, *Transactions of Institution of Naval Architects,* 1873, p. 179. The pendulum should have sufficient weight and the arm carrying the pen may be about 4 ft. long. If the cylinder be fitted with a clock recording the time the natural period of the ship will also be obtained.

The righting lever is GZ as before ; if V be the volume of displace- ment, and *υ* that of either wedge, then

V×BR≡υ×Λ⅛

also

GZ = BR-BG sin *Θ∖*

whence the righting moment or

W×GZ =∖V ^⅛-BG sin 9 ! ·

This is termed Atwood’s formula. Since BG, V and W are usually known, its application to the computation of stability at various angles and draughts involves only the determination of t>×ΛιZ⅛. A convenient method of obtaining this moment was introduced by F. K. Barnes and published in *Trans. Inst. N.A.* (1861). The steps in this method were as follows: (*a*) assume a series of trial water lines at equal angular intervals radiating from S' the intersection of the upright water line with the middle line plane; *(b)* calculate the volumes of the various immersed and emerged trial wedges by radial integration, using the formula *y=i^dφfr"-dx,*

where *r*, *Φ* are the polar co-ordinates of the ship’s side, measured from S' as origin, and *dx* an element of length; *(c)* estimate the moment of transference of the same wedges parallel to the particular trial water line by the formula

*υ × h1h2* = i J^cos(0 — *φ)dφfridx,*

adding together the moments for both sides of the ship; and *(d)* add or subtract a parallel layer at the desired inclination to bring the result to the correct displacement. The true water line at any angle is obtained by dividing the difference of volume of the two wedges by the area of the water plane (equal to *frdx,* for both sides) and setting off the quotient as a distance above or below the assumed water line according as the emerged wedge is greater or less than the immersed wedge. The effect of this “ layer correction ” on the moment of transference is then allowed.

The righting moment and the value of GZ are thus determined for the displacement under consideration at any required angle of heel.

A different method of obtaining the righting moments of ships at large angles of inclination has prevailed in France, the standard investigation on the subject being that of M. Reech first published in his memoir on the “ Construction of Metacentric Evolutes for a Vessel under different Condi­tions of Lading” (1864). The principle of his method is dependent on the following geo­metrical properties:—

Let B,t B\* (fig. 8) be the centres of buoy- ancy corresponding to two water lines W'L', ∖V\*Lr inclined at angles *Θ, θfi-dθ,* to the original upright water line WL, *dθ* being small; and let fι, *gì* be the centres of gravity of the equal wedges W'TW', L'TL'. The moment of either wedge about the line *gιg2* is zero, and the moments of W'L'A and of W\*L\*A about gιg2 are therefore equal; since these volumes are also equal, the per- pendicular distances of B' and from gιg2 are equal, or B'B\* is parallel to g1g2∙

The projection on the plane of inclination of the locus of the centre of buoyancy for varying inclinations with constant displacement is termed the *curve of buoyancy,* a portion BB'Bff of which is shown in the figure. On diminishing the angle *dθ* indefinitely so that B\* approaches Bz to coincidence, the line B'B" becomes, in the limit, the tangent to the curve BB'B\*, and gιg2 coincides with the water line W'L'; hence the tangent to the curve of buoyancy is parallel to the water line.

Again, if the normals to the curve at B', B" (which are the verticals corresponding to these positions of the centre of buoyancy) intersect at M', and those at B", B"' (adjacent to B") at M", and so on, a curve may be passed through M', M", . . . , commencing at M, the meta- centre. This curve, which is the evolute of the curve of buoyancy, is known as the *metacentric curve,* and its properties were first