centre of gravity, or, as it is termed, the centre of flotation, the curve of flotation will be the locus of the projections of the centres of flotation on the plane of the figure, which curve touches each water- line.

From consideration of the slope of a ship’s side around the peri- phery of a water-line, Dupin obtained the following expression for p', the radius of curvature of the curve of flotation, pz = *~~fÿ\* tan a,. ds~~* j both sides,

p area of water-plane t

where *ds* is an element of the perimeter, *a* the inclination of the ship’s side to the vertical, and *y* its distance from the longitudinal axis through the centre of flotation. M. Emile Leclert, in a paper read at the Institution of Naval Architects, 1870, proved the equivalence of the above formula to the two following, which are known as Leclert’s Theorem:

p' = p+V^andp' = ^,

where I and V are respectively the moment of inertia of the water­plane and the volume of displacement, and *p* is the radius of the curve of buoyancy or B'M'. Independent analytical proofs of the formulae were given in the paper referred to; and *(Trans. I.N.A.,* 1894) a number of elegant geometrical theorems in connexion with stability, given by Sir A. G. Greenhill, include a demonstration of Leclert’s Theorem as follows (in abbreviated form) :

Let B, B1 (fig. 17) be the centres of buoyancy of a ship in two consecutive inclined positions, and F, F1 the corresponding centres of flotation. Draw normals BM, B1M, meeting at the pro-metacentre M, and FC, F1C, meeting at the centre of curva­ture C. Produce FB, F1B1 to meet at 0; join OM, MC.

Then BM, CF and B1M, CF1 are respectively parallel, and ultimately also BB1, FF1; hence the triangles MBB1, CFF1 are similar and

BM-BBι OB CF -FF7≈DF' so that O, M and C are collinear.

If the displacement V be now increased by dV, changing B to B', and M to M', then since the added displacement *dV* may be supposed concentrated at F, B' will lie on OBF, and it may be shown similarly as before that M' lies on OC. Further, considering the transference of moments, BB'×V = BF×dV.

Draw MED parallel to BF, then

rfV-BBz ME MzE *dp* V BF ^MD' CD·\_7^:

*• dp p' — p f - dp*

∙∙3v--v-orp =p+v3v-

giving Leclert’s first expression ; also, since *p =*

di *l -⅝, dp t* 3V=P÷vjy=P.

which is Leclert’s second expression for *p'.*

The value of *pt* at the upright can be obtained from the metacentric diagram by the following simple construction. Let M and B be the metacentre and the centre of buoyancy for a water-line WL on the metacentric diagram (fig. 18) ; draw the tangent to the B curve meeting WL at Q, and through Q draw QR to meet MB and parallel to the tangent to the M curve at Μ. Let BP = *h*, and area of water-line be A. Then V V

PQ = A cot 0=⅛2tsA\* also.

MR = BM — (BP+PR) =p—(tan Θψtan ≠). If D be the draught,

*λ , dp . dp*

tan 0-∣-tan φ= —= A∙^γ,

whence

MR=p+V^=p'

the curve of flota- tion being concave upwards if R is below M.

For moderate inclinations from the upright, the buoy- ancy of the added layer due to a small additional submer- sion will act through the centre of curva­ture of the curve of flotation ; this point may be regarded as that at which any additional weight will, on being placed on a ship, cause no difference to the values of the righting moment at moderate angles of inclination. The curve of flotation, therefore, and its evolute bear similar relations to the increase or decrease of the stability of a ship due to alteration of draught, as the curves of buoyancy and of pro-metacentres do to the actual amount of the stability.

The curve of flotation resembles the curve of buoyancy in that not more than two tangents can be drawn to it in any given direction, but it differs in that its radius of curvature can become infinite or change sign. It contains a number of cusps determined by *p'=^=0.* These occur in an ordinary ship-shape body at positions: (1) at or near the angles at which the deck is immersed or emerged (four in number); and (2) at or near the angles 90o and 270°. There are, therefore, six cusps in the curve of flotation of an ordinary ship; they are shown in figs. 15 and 16 by the points F2, F3, F4, F6, F7, F8.

The following relations between the curves of buoy- ancy and of pro-metacentres and the curve of statical stability are of interest, and enable the former curves to be constructed when the latter have been obtained. If GZ', GZ' (fig. 19) are the righting levers corresponding to inclinations *θ, θ* 4- *dθi* where *dB* vanishes in the limit; B', B\*, the centres of buoyancy, M' the pro- metacentre; produce GZ' to meet B'M' in U.

Then, neglecting squares of small quantities, d(GZ')=Z'U~M'Z'.dÖ,

or vertical distance of M' above

Also M'B' = M'B\*j hence

Z,B\*-Z'B' = MZ'-MZ' = Z'U = GZ'.d0, or

d(B'Z')

*gz"-dθ~>*

*i.e.* the vertical distance (B'Z') of G over B is equal to*fGZ.dθ.*

It follows that by differentiating the levers of statical stability and finding the slope at each ordinate the vertical distance of M' over G is obtained, and M' may be plotted by setting up this value from Z' above GZ' drawn at the correct inclination ; also that by integrating the curve of statical stability and finding its area up to any angle, the vertical separation of G and B' is obtained, and B' may be plotted by setting down this value increased by BG below Z'.