where Ix, Iy are the principal moments of inertia of the water-plane. Hence

0 τ ·

x= —y∙Itf∙s1n *φ;*

*ø*

*y≈* ÿ·Ix·cos φ;

02

2= ⅜y(lχcos2φ+Ivsin2φ).

Eliminating *Ô* and *φ,* the locus of the centre of buoyancy for small inclinations of the ship becomes the elliptic paraboloid— x2 y2

22.=Uv+i√v'

The equation to the indicatrix referred to axes parallel to Bx, By is therefore

I7v+i⅛=constantι

and the indicatrix is therefore similar and similarly situated to the momental ellipse of the water-plane, and the surface of buoyancy is everywhere synclastic and concave to all points within it. The quantities Iy∕V and Ix∕V are evidently equal to BMx and BMy (referring to inclinations about Oy and *Ox* respectively); and the indicatrix and momental ellipse become

Bfc+⅛=constant∙ The angle *ψ* that BB2 (the projection of BB' on the plane of the indicatrix) makes with xO is given by

*y* Iχ

tan ≠ = — 7=r. cot *Φ;*

X 1 y hence the direction is conjugate to that of the axis of rotation with respect to the indicatrix.

This is illustrated in fig. 25, where the ellipse shown is the indicatrix; OPx' the axis of inclination, OQ the conjugate radius, and ORMy' the per- pendicular on the tangent. Draw QN parallel to OM to meet OP. The tri- angle OMQ is similar to BB1B2; and they can be made equal by giving a suitable value to the constant in the indicatrix equation. In that case QN is the projection on the plane of the figure of the normal to the surface at B1, and the shortest distance between\* the normals at B and Bl is equal to ON = MQ = Pθ

BιBa≡=y-, since ON or the axis of inclination is perpendicular to them both. Also, the length B'M of the normal at B' intercepted between Br and the foot of the common perpendicular is equal to ON

≈p since *θ* is the angle between the normals at B and B'; it follows ‰ d/w BBj ↑xf

that BM=-y=^,

an expression analogous to that obtained before for the case of small inclinations in the direction of the principal axes of the water- plane. It is worthy of note that the radius of curvature *p* of the normal section of the surface of buoyancy through Oy' is, in general,

OM2 less than BM; the latter being equal to ——, and *p* being equal 2Z

OR2 to *p* is also obtainable by Euler’s equation—

I cos2√> 1 sin2⅜ . p~BMz BM∕

becoming equal to BM for inclinations about the principal axes. Similarly the radius of curvature of the normal section through Q is, in general, greater than BM.

If the centre of gravity G of the ship is coincident with B, the arm jχ'

of the righting couple is OM or γ··0; and there is also a couple of P

lever ON or *γ∙0* in a perpendicular vertical plane. The resultant couple lies in a plane containing OQ, having a lever equal to oq ∞ *⅞∙∖jb'1+p5 oτ ⅛∙∖pcos 2ψ+1-'2 sin 2φ∙*

In the general case when G is situated at a distance *a* above B, the righting lever becomes and the perpendicular couple is unaltered. The resultant couple can be readily found, but in this case it bears no simple relation to the indicatrix, as before ; it may be shown, however, that the plane of the couple is conjugate to the axis of inclination with respect to the confocal ellipse

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *φ* | 0° | 1o | 5° | 10° | 20° | 30° | 40° | 50o | 60° | 70° | 80° | 90° |
| GM | 4' | 4∙1 | 7' | 16 | 50·4' | 103' | 168' | 237 | 300 | 354 | 388 | 400' |
| α | 0° | 60° | 78·5 | 76·8° | 68·5° | 59° | 49∙3° | 39·5° | 29∙7° | 19∙8° | 9·9° | O° |
| *Ψ* | 90° | 29∙0° | 6∙5° | 3∙2° | 1∙5° | 1∙0° | 0.7° | 0∙5° | 0∙3° | 0∙2° | 0.1° | 0° |

x2 y2

ï — = constant.

*∖∕~α ^-a*

, In the case when GM =0, the ship being in neutral equilibrium for that direction of inclination, the resultant couple is parallel to the axis Ox', *i.e.* perpendicular to the plane of the indicatrix.

Numerical values of the metacentric height GM, the angle of obliquity α or QOM (equal to tan^~1~~\* , J~~ ) and the angle ≠ are given in the following table for a ship whose transverse GM is 4 ft., longi­tudinal GM 400 ft., and BG 10 ft.:—

The greatest angle of obliquity (a) occurs in this case when *φ* is about 5¾° and the plane of the couple is nearly coincident with the middle line plane for all angles of *φ* greater than about 30°. It follows that if a weight is moved obliquely across the ship the axis of rotation is approximately longitudinal, except when the line of movement is nearly fore and aft; and in the latter case a small deviation from a fore and aft direction produces a large change in the position of the axis of rotation.

The direction of the axis of rotation is above expressed with reference to the position of the inclining couple in relation to the indicatrix of the surface of buoyancy; as, however, the couple is assumed small, the direction of the axis and the amount of inclination may equally be obtained by resolving the couple in planes perpendicular to the principal axes and superposing the separate inclinations produced by its components.

It has been shown above that the positions of equilibrium are found by drawing all possible normals to the surface of the buoyancy, and the condition for stability for an inclination in any direction is that the centre of gravity shall lie below the corresponding metacentre. The height of the metacentre varies with the moment of inertia of the water-plane about the axis of inclination, and the maximum and minimum heights are associated with the maximum and minimum moments of inertia, which again correspond to inclinations about the least and greatest axes of inertia respectively. If the centre of gravity lies below the lowest position of the metacentre (the transverse metacentre in the case of a ship when upright) the equilibrium is stable for all inclinations, and the condition is referred to as one of *absolute. stability* ; if it lies above the highest meta- centre, the condition is one of *absolute instability;* if it lies between the highest and lowest metacentres, the condition is one of *relative stability,* the ship being stable for inclinations about a certain set of axes, and unstable otherwise.

The foregoing remarks apply to a vessel whose axis of inclination is fixed so that the component couple perpendicular to the plane of inclination is resisted. If, on the other hand, the vessel is free to move in all directions the resultant couple does not in general tend to restore the original position of equilibrium, although the com­ponent in the plane of inclination complies with the conditions above stated for absolute stability. If *m1* and be the greatest and least values of GM, the ratio of the component couples perpendicular to and in the plane of inclination, or tan α (fig. 25), is greatest when tan *Φ = A∣~^'f* and then tan α≡^⅛—If *mi∕mi* be small, this *∖ m \*\* m1m2*

ratio is large, being equal to 4∙95 in the numerical example above. In such cases the extent of the movement that can result from a small initial disturbance cannot be readily determined by a statical method, but the investigation of the work done in moving the vessel from one position to another appears to meet this difficulty.

This process is employed by Μ. Guyon in his *Théorie du navire,* the stability of a ship in any condition being treated throughout from the dynamical standpoint. He proved that:—

1. For changes of displacement, without change in inclination, the potential energy of a system consisting of a floating body and the water surrounding is a minimum when the weight of the body is equal to its displacement.

2. For changes of direction, without change of displacement, the potential energy of the system is equal to the weight of the body, multiplied by the vertical resolute of BG; when this distance is a minimum or a maximum the stability is respectively stable or unstable. A statical proof of this has been given in the two- dimensional case.

The potential energy is thus equal to the dynamical stability