respectively. Froude gave his reasons for expecting the resistance to vary partly as the first and partly as the second power of the angular velocity. The latter part he considered would be due to surface­friction and the head resistance of keels and deadwood, and the former to the resistance caused by the creation of a small wave at each roll, which, by travelling away from the ship, would cause dissipation of energy. Froude’s views have been confirmed by the accuracy with which the expression — j^ = α.θ⅛δ.θi may be made to fit the curve of extinction of practically any ship by the judicious selection of the coefficients *a* and *b*. Μ. Bertin has, however, preferred an expression equivalent to while other French investigators have preferred an expression equivalent to *~<∏i~ σθ∙*

On substituting the value of *a* in equation (7) it becomes—

β=Aeτsin . . . (8)

a simplified form of the equation for resisted rolling when the coefficient *b* is neglected.

For the “ Revenge ” the following equations represent the curves of extinction given in fig. 29 :

For *deep draught :*

\* t∕Θ

without bilge keels—= \*01230-}-·0025Θ2

\* ⅛7Θ

with „ „ ~^≈∙°65 Θ + ∙oi7 θ2

For *light draught :*

without bilge keels—^ = ∙o15 Θ + ∙oo28Θ2

with „ „ “^ = \*084 Θ + ∙oι9 Θ2

(θ in all cases being measured in degrees and not in circular measure).

The large increase in the *b* coefficient after bilge keels had been fitted has given rise to considerable discussion. Mr R. E. Froude had experimented with a deeply submerged plane oscillating in water, and he found that at a speed of 1 foot per second the resistance per square foot was 1∙6lb. Using this figure to calculate the work per swing from an extreme angle of 6°, the head-resistance of the bilge keels is found to account for about one-fourth the energy lost in a single swing due to the increased value of the *b* coefficient in the above formula. The energy abstracted in this particular case is thus about four times greater than the theoretical head-resistance of the bilge keels. This discrepancy has been observed in many cases, and it appears that when bilge keels are added to a ship they become effective, not merely as flat surfaces moving with the ship and experiencing direct resistances, but also by indirectly influencing the stream-line motions which would exist about the oscillating ship, if there were no bilge keels. Another cause of the difference is that the bilge keels during the early portion of the swing set into motion a large mass of water, only a small proportion of whose energy is returned to the ship towards the end of the roll. This condition is accentuated when the vessel is in motion ahead, and owing also to the increase of other resistances at high speeds, a more rapid extinction is then obtained. It appears from experiments made on H.M.S. “ Revenge ” and on a torpedo boat destroyer that the extinction at a given angle of roll is given by a linear formula *-dθ≈α-∖-β∖',* where *a* and *ß* are coefficients independent of the speed V.@@1

Froude attacked the problem of resisted rolling in an inverse manner, endeavouring to ascertain “ what wave-series is required to keep the given ship at a given range of steady rolling with any assigned period, including the effect of resistance.” Subsequently he treated the problem in a direct manner by the process of "graphic integration,” an exact method of determining the motion of a ship, the elements of the ship’s rolling in still water and the wave-series acting upon her being given.@@2 Some interesting developments of the process were made by Sir William White in a paper read before the Inst. Nav. Arch. in 1881 on the “ Rolling of Sailing Ships,” in which the action of the wind on the sails and the variation of the virtual weight of the ship on the wave are included. The effect of wind- pressure in heeling a ship is very much greater when she is at the crest of a wave than when she is at the trough, because her virtual weight is less. This must be taken into account when dealing with sailing vessels; the reduction of virtual weight, and therefore of righting moment, at the crest of a wave being very considerable, although the heeling moments due to the wind suffer no such reduction.

The differential equation for rolling among waves including the effect of resistances varying as the first power of the angular velocity *is- ∖V<\*<Pθ . vdθ .... ∕ ω* . π.∖

T^Λs+κ2z+w"∙ V-θιsιnT1√ =o’ which becomes on substitution (K being expressed in terms of *a*)— *d?6 , 2adθ* 7Γ2 √ q . π .

Φ+T7i÷T≈β =5p ■θ, s,n *T‘·*

The general solution is—

~α∕ \_\_\_\_\_

0≡Ac sin (∙ψ,^ι--~λ+∕3) +AiΘj sin (,ψj-» · (9)

where

1 ∕ T2∖2,4α2T2 1λ 2αTT1

Aι2- V“T?/ + ιriT1≈a"d ft~tan x(Tιs-T2) and A and *ß* are arbitrary.

The first term represents a free oscillation of the ship, which in time dies out, leaving a forced oscillation in the period of the waves. From observations on rolling, however, it is found that, owing to the departure from exact uniformity in the waves encountered, a ship seldom, if ever, completely forsakes her own natural period of rolls; for each slight alteration in the wave period T1 introduces afresh terms involving the free oscillations of the ship. In the synchronizing conditions where T = T1, the forced oscillation is represented by ιr ’

0=-^θ>cosT∙

the amplitude being limited entirely by the resistance; the phase is before that of the wave slope. The vessel is then upright in mid-height, and inclined to its maximum angle on the crest and in the hollow of the wave. The maximum amplitude Θ is given by ∣∙Θι=α.Θ. Since the right-hand term represents the decrement of roll due to resistance, the left-hand side must represent the increment of roll due to the wave in this synchronizing steady motion. If this latter relation be assumed to hold when the resist- ance to motion is represented by the more general decremental equation, then the maximum amplitude Θ is given by

-Θι==α.Θψδ.Θi.

In 1894 and 1895 M. Bertin, at the Institution of Naval Architects, extended this relation to cases in which T1 is not equal to T, obtaining at the same time not simply the angles of steady rolling for these cases, but the maximum angles passed through on the way to the steady condition; to these maximum angles he gave the name of “ apogee ” rolls.

In 1896, at the Institution of Naval Architects, Mr R. E. Froude investigated the probable maximum amplitude of roll under the influence of a non-synchronous and non-harmonic swell. He imagined three identical ships, A, B and C, the first rolling in still water, and the two others placed in the same swell assumed recurrent in period 2T1, but not necessarily harmonic. Assuming resistance to vary as then denoting the vessels by suffixes, the effective wave slope by 0ι, and constants by K, K' and K",

<220a 1 tλ<Z0a I tλm \_

^+κ^+κ‰=κ,eι! ⅛c+⅛c+κ‰≡K'βl.

@@@1 See papers on this subject read before the Institution of Naval Architects in 1900 by Professor Bryan and in 1905 and 1909 by Mr A. W. Johns.

@@@2 See *Trans. Inst. Naval Arch.,* 1875.