giving a compression, and since the slope is steepest there, or *— dy∣dx* a maximum, the compression is also a maximum there. When the displacement is represented by AH'BK'C the particles on each side of B are displaced from it, giving an extension, and since the slope is again the steepest, the extension is a maximum.

At the loops, for instance at H, the displacement is a maximum. The tangent to the displacement curve is always parallel to the axis, that is, for a small distance the successive particles are always equally displaced, and therefore always occupy the same volume. This means that at the loops while the motion is greatest there are no pressure changes.

We have now to select such portion of this system as will suit the conditions imposed by any actual pipe. There are three distinct types, which we will consider in succession.

I. *Pipe Closed at One End, Open at the Other.—*At the closed end there is no motion, for the pressure always constrains the air to remain in contact with the end. The closed end is therefore a node. At the open end, as a first approximation to be corrected later, there are no pressure changes, for any tendency to excess can be relieved by immediate expansion into the outer air, and any tendency to defect can be filled up by an inrush from the outer air. The open end is therefore a loop. It is to be noted that the exciter of the vibrations is in general at the open end, and that the two trains forming the stationary system consist of the direct waves from the exciter travelling into the tube, and the waves reflected back from the closed end.

In fig. 27 we may have the length AH occupying the tube. In this case AH = ¼λι=*l*, the length of the tube, and the frequency n1≡U∕λι = U∕4*l*. But we may also have a shorter wave-length λ2 such that the length AK occupies the tube. In this case AK = ¾λ2=*l*, and the frequency n2 = U∕λ2 = 3U∕4*l*. With a still shorter wave-length λ3 we may have the length AL occupying the tube and AL = 5/4λ2 = *l*, and the frequency n3 = U∕λ8=5U∕4*l*, and so on, as we take succeeding l∞ps for the open end.

In fig. 28 are represented the stationary wave systems of the first four modes, and any of the succeeding ones are easily drawn.

The reader will be able to make out the simultaneous motions and pressures at various points. It is obvious that the nodes are alternately in compression and extension, or vice versa, and that for ¼λ on each side of a node the motion is either to it on both sides or from it on both sides.

The first mode of vibration gives the “ fundamental tone," and the succeeding modes are termed “ overtones.” The whole series forms the series of odd harmonics. A “ stopped pipe ” in an organ is a pipe of this type, and both the fundamental and the overtones may occur simultaneously when it is blown.

We may illustrate the successive modes of vibration by using as pipe a tall cylindrical jar, and as exciter a vibrating tuning-fork held over the mouth. The length of the pipe may be varied by pouring in water, and this is done until we get maximum resonance of the pipe to the fork. Thus if a fork U*t*3 = 256 is used, the length of pipe for the fundamental at 0o C. is about 33,000∕4×256 = 33 cms. If a fork Sol4 = 768 is used the pipe resounds to it according to the mode of the first overtone. If the temperature is *t*0 the length for given frequency must be increased by the factor 1 +0·00184*t*.

*Correction to Length at the Open End.*—The approximate theory of pipes due to Bernoulli assumes a loop at the open end, but the condition for a loop at the open end, that of no pressure variation, cannot be exactly fulfilled. This would require that the air outside should have no mass in order that it should at once move out and relieve the air at the end of the pipe from any excess of pressure, or at once move in and fill up any defect. There are variations, therefore, at the open end, and these are such that the loop may be regarded as situated a short distance outside the end of the pipe. It may be noted that in practice there is another reason for pressure variation at the end of the pipe. The stationary wave method regards the vibration in the pipe as due to a series of waves travelling to the end and being there reflected back down the pipe. But the reflection is not complete, for some of the energy comes out as waves; hence the direct and reflected trains are quite equal, and cannot neutralize each other at the loop.

The position of the loop has not yet been calculated for an ordinary open pipe, but Lord Rayleigh has shown *(Sound,* ii. § 307) that for a cylindrical tube of radius R, provided with a flat extended flange, the loop may be regarded as about 0∙82 R, in advance of the end. That is, the length of the pipe must be increased by 0∙82 R before applying Bernoulli’s theory. This is termed the “ end correction.”

Using this result Rayleigh found the correction for an unflanged open end by sounding two pipes nearly in unison, each provided with a flange, and counting the beats. Then the flange was removed from one and the beats were again counted. The change in virtual length by removal of the flange was thus found, and the open end correction for the unflanged pipe was 0∙6 R. This correction has also been found by David James Blaikley by direct experiment *(Phil. Mag.,* 1879, 7, p. 339). He used a tube of variable length and determined the length resounding to a given fork, (1) when the closed end was the first node, (2) when it was the second node. If these lengths are *l*1 and *l*2, then *l*2-*l*1 = ½λ and ½*(l2 —l1*)-*l1* is the correction for the open end. The mean value found was 0∙576 R.

2. *Pipe Open at Both Ends.—*Each end is a loop. We must there- fore select a length of fig. 27 between two loops. The fundamental mode is that in which H and K represent the ends of the pipe. In this case HK = ½λ1=*l*, and the frequency is *n*1 = U∕λι = U∕2*l*. There is a node in the middle. In the next mode H and L represent the ends and HL=λ2=*l* and n2 = U∕λ2 = 2U∕2*l*. In the third mode HM=3/2λ3=*l* and n3=U∕λ3 = 3U∕2*l*, and so on.

In fig. 29 are represented the stationary wave systems of the first four modes. The whole series of fundamental and overtones gives the complete set of harmonics of frequencies proportional to 1, 2, 3, 4, ..., and wave-lengths proportional to 1, ½ ⅓, ¼... .

A metal or brass tube will serve as such a pipe, and may be excited by a suitable tuning-fork held at one end. To obtain the virtual length we must add the correction for each open end, probably about 1∙2 radius. If the frequency is 256 the corrected length for the fundamental is about (33,000/2 ×256) (1+∙00184*t*) at t0. The pipe will also resound to forks of frequencies 512, 768, 1024 and so on.

An open “ flue” organ pipe is of this type. The wind rushing through the slit S (fig. 30) maintains the vibration in a way to be discussed later, and the opening O makes the lower end a loop.

The modes of vibration in an open organ pipe may be exhibited by means of Koenig’s manometric flames *(Phil. Mag.,* 1873, vol. 45). \* The pipe is provided with manometric flames at its middle point, and at one-quarter and three-quarters of its length. When the pipe is blown softly the fundamental is very predominant, and there is a node at the middle point. The flame there is much