extends radially and in the ratio σ, known as Poisson’s ratio, which in metals is not far from ¼. Let us suppose that the rod is circular, of radius *r,* and that the radial displacement of the surface is *η.* The longitudinal extension is *dy∣dx,* and therefore the radial contraction is *nlr = σdyldx.* If then y = α sin — (x—UZ), *η = 27r°ra* cosy-(x — U0. If *r* is of the order of λ, *η* is of the order of *y;* and the kinetic energy of the radial motion is of the same order as that of the longitudinal motion. But our investigation entirely leaves this out of account, and is therefore faulty. In fact, the forces are then no longer parallel to the axis. There are shears of the order *dn∣dx* and the simple Young’s modulus system can no longer be taken to represent the actual condition (see Rayleigh, *Sound,* i. § 157). But keeping r∕λ small we may as before form stationary waves, and it is evident that the series of fundamental and overtones will be just as with the air in pipes, and we shall have the same three types—fixed at one end, free at both ends, fixed at both ends—with fundamental frequencies respectively

The overtones will be obvious.

For an iron wire Y/p is about 1012∕4, so that for a frequency of 500 in a wire fixed at both ends a length about 5 metres is required. If the wire is stretched across a room and stroked in the middle with a damp cloth the fundamental is easily obtained, and the first harmonic can be brought out by stroking it at a quarter the length from one end. A glass or brass rod free at both ends may be held by the hand in the middle and excited by stroking one end outwards with a damp cloth. If it is clamped at one-quarter and three- quarters of the length from the ends, and is stroked in the middle, the first harmonic sounds.

Young’s modulus may be obtained for the material of a rod by clamping it in the middle and obtaining the frequency of the funda- mental when Y=4∕2n2p.

The value thus obtained is generally appreciably greater than that obtained by a statical method in which the rod is pulled out by an applied tension.

Rods of different materials may be used as sounders in a Kundt’s dust tube, and their Young’s moduli may be compared, since:—

velocity in rod = velocity in air × length of rod/distance between dust-heaps.

*Torsional Vibrations of Rods and Wires.—*The velocity of propagation of a torsional disturbance along a wire of circular section may be found by the transfer of momentum method, remembering that we must now replace linear momentum by angular momentum. Let the disturbance be supposed to travel unchanged in form from left to right with velocity Û. Now suppose that the wire or rod is moved from right to left with velocity U. The disturbance is then fixed in space. Let A be a point in the disturbance and B a point in the undisturbed portion. The condition of the matter between A and B remains constant, though fresh matter keeps coming in at B and an equal quantity leaves at A. Hence the angular momentum of the part between A and B remains constant, or as much enters at B as leaves at A. But at B there is no torsion, and no torsion couple of one part of the wire on the next. So that no angular momentum enters at B, and therefore on the whole none loaves at A. The transfer of angular momentum through A is of two kinds—first, that due to the passage of rotating matter, and, secondly, that due to the couple with which matter to the right of A acts upon matter to the left of A. The mass of matter moving through A per second is pπα2U, where *a* is the radius of the wire and *p* is its density. If *θ* is the angle of twist, the angular velocity is *dθ∣dt.* The radius of gyration of the section is ½*a*2. Hence the angular momentum conveyed per second outwards is *⅛pπa4Udθ/dt.* The couple due to the twist of a wire of length *l* through *ψ* is G = ½*nτraiφ∕l,* and we may put *φ∣l≈dθ∣dx.* Since no angular momentum goes out on the whole

2 *nπa\*dβ∣ dx -∣- ⅛pπa4ljd0∕dt* = o. (33)

But the condition of unchanged form requires that the matter shall twist through *(flθ∣dx)dx* while it is travelling over *dx, i.e.* in time dx∕U.

dø *dx dθ j dß τxdθ τhen τι* π=-2idxor3∕ = -⅛∙

Substituting in (33) we get

U2 = n∕p. (34)

If we now keep the wire at rest the disturbance travels along it with velocity U=√(n∕p), and it depends on the rigidity and density of the wire and not upon its radius.

It is easy to deduce the modes of vibration from stationary waves as in the previous cases. If a rod is clamped at one end and free at the other, the fundamental frequency is (1∕*l*)√ (*n*∕p). For iron *η/p* is of the order 1011, so that the frequency for a rod 1 metre long is about 3000. When a cart wheel is ungreased it produces a very high note, probably due to torsional vibrations of the axle.

The torsional vibrations of a wire are excited when it is bowed. If small paper rings are put on a monochord wire they rotate through these vibrations when the wire is bowed.

*Transverse Vibrations of Bars or Rods.—*When a bar or rod is of considerable cross-section, a transversal disturbance calls into play forces due to the strain of the material much more important than the forces due to any tension which is ordinarily applied. The velocity of a disturbance along such a bar, and its modes of vibration, depend therefore on the elastic properties of the material and the dimensions of the bar. We cannot investigate the vibrations in an elementary manner. A full discussion will be found in Rayleigh’s *Sound,* vol. i. ch. 8. We shall only give a few results.

The cases interesting in sound are those in which (1) the bar is free at both ends, and (2) it is clamped at one end and free at the other.

For a bar free at both ends the fundamental mode of vibration has two nodes, each o∙224 of the length from the end. The next mode has a node in the middle and two others each 0∙132 from the end. The third mode has four nodes o∙o94 and o∙357 from each end, and so on. The frequencies are nearly in the ratios 32:52:72. . . . Such bars are used in the harmonicon.

When one end is clamped and the other is free the clamped end is always a node. The fundamental mode has that node only. The next mode has a second node 0∙226 from the free end; the next, nodes at 0∙132 and o∙5 from the free end, and so on. The frequencies are nearly in the ratios 1:6·25:17·5. Such bars are used in musical boxes and as free reeds in organ pipes.

The most important example of this type is the tuning-fork, which may be regarded as consisting of two parallel bars clamped together at the base. The first overtone has frequency 6∙25 that of the fundamental, and is not in the harmonic series. If the fork be mounted on a resonance box or held in front of a cavity resounding to the fundamental and not to the first overtone, the fundamental is brought out in great purity.

*Vibrations of Plates.—*These are for the most part interesting rather from the point of view of elasticity than of sound. We shall not attempt to deal with the theory here but shall describe only the beautiful mode of exhibiting the regions of vibration and of rest devised by E. F. F. Chladni (1756-1827). As usually arranged, a thin metal plate is screwed on to the top of a firm upright post at the centre of the plate, which is horizontal. White sand is lightly scattered by a pepper-box over the plate. The plate is then bowed at the edge and is thrown into vibration between nodal lines or curves and the sand is thrown from the moving parts or ventral segments into these lines, forming "Chladni’s figures.” The development of these figures by a skilful bower is very fascinating. As in the case of a musical string, so here we find that the pitch of the note is higher for a given plate the greater the number of ventral segments into which it is divided ; but the converse of this does not hold good, two different notes being obtainable with the same number of such segments, the position of the nodal lines being, however, different.

The upper line of annexed figures shows how the sand arranges itself in three cases, when the plate is square. The lower line gives the same in a sort of *idealized* form. Fig. 38, 1, corresponds to the lowest possible note of the particular plate used; fig. 38, *2,* to the *fifth* higher; fig. 38, 3, to the *tenth* or octave of the *third,* the numbers of vibration in the same time being as 2 to 3 to 5.

If the plate be small, it is sufficient, in order to bring out the simpler sand-figures, to hold the plate firmly between two fingers of the same hand placed at any point where at least two nodal lines meet, for instance the centre in (1) and (2), and to draw a violin bow downwards across the edge near the middle of a ventral segment. But with larger plates, which alone will furnish the more complicated figures, a clamp-screw must be used for fixing the plate, and, at the same time, one or more other nodal points ought to be touched with the fingers while the bow is being applied. In this way, any of the possible configurations may be easily produced.

By similar methods, a circular plate may be made to exhibit nodal lines dividing the surface by diametral lines into four or a greater, but always *even,* number of sectors, an odd number being incompatible with the general law of stationary waves that the parts of a body adjoining a nodal line on either side must always vibrate oppositely to each other.

Another class of figures consists of circular nodal lines along with diametral lines (fig. 39).