is due to reduction of the end correction. When the air rushes out from one pipe, it has not to force its way into the open air, but finds a cavity being prepared for it close at hand in the other pipe, and so the extensions and compressions at the ends are more easily reduced. Even the longer pipe may be effectively shorter than the corrected shorter pipe when sounding alone.

*Beats.*

When two notes are not quite in unison the resulting sound is found to alternate between a maximum and minimum of loudness recurring periodically. To these periodical alternations has been given the name of *Beats.* Their origin is easily explic- able. Suppose the two notes to correspond to 200 and 203 vibrations per second; at some instant of time, the air particles, through which the waves are passing, will be similarly displaced by both, and consequently the joint effect will be a sound of some intensity. But, after this, the first or less rapidly vibrating note will fall behind the other, and cause a diminution in the joint displacements of the particles, till, after the lapse of one- sixth of a second, it will have fallen behind the other by half a vibration. At this moment, therefore, opposite displacements will be produced of the air particles by the two notes, and the sound due to them will be at a minimum. This will be followed by an increase of intensity until the lapse of another sixth of a second, when the less rapidly vibrating note will have lost another half-vibration relatively to the other, or one vibration reckoning from the original period of time, and the two component vibrations will again conspire and reproduce a maximum effect. Thus, an interval of one-third of a second elapses between two successive maxima or beats, and there are produced three beats per second. By similar reasoning it may be shown that the number of beats per second is always equal to the difference between the numbers of vibrations in the same time corresponding to the two interfering notes. The more, therefore, these are out of tune the more rapidly will the beats follow each other.

The formation of beats may be illustrated by considering the disturbance at any point due to two trains of waves of equal ampli­tude *a* and of nearly equal frequencies *n*1 *n*2. If we measure the time from an instant at which the two are in the same phase the resultant disturbance is

*y=a* sin 2τw√+α sîn 2tm√

= 2α cos tγ(mi-W2)Z sin 7r(w1-∣-n2)Z, which may be regarded as a harmonic disturbance of frequency (n1+w2)∕2 but with amplitude *2a* cos tγ(mi-*nψt* slowly varying with the time. Taking the squares of the amplitude to represent the intensity or loudness of the sound which would be heard by an ear at the point, this is

4α2 cos2 π(nι-«2)Z

= 2α2{i+cos 2τr(wι-W2)Z),

a value which ranges between 0 and 4α2 with frequency wi — *n^.* The sound swells out and dies down *n*1-*n*2 times per second, or there are *n*1—*n*2 beats per second. If, instead of considering one point in a succession of instants, we consider a succession of points along the line of propagation at the same instant, we evidently have waves of amplitude varying from *2a* down to o, and then up to *2a* again in distance U∕(*n*1-*n*2)*.*

The phenomena of beats may be easily observed with two organ- pipes put slightly out of tune by placing the hand near the open end of one of them, with two musical strings on a resonant chest, or with two tuning-forks of the same pitch mounted on their resonance boxes, or held over a resonant cavity (such as a glass jar), one of the forks being put out of tune by loading one prong with a small lump of beeswax. In the last instance, if the forks are fixed on one solid piece of wood which can be grasped with the hand, the beat will be actually felt by the hand. If one prong of each fork be furnished with a small plain mirror, and a beam of light from a luminous point be reflected successively by the two mirrors, so as to form an image on a distinct screen, when one fork alone is put in vibration, the image will move on the screen and be seen as a line of a certain length. If both forks are in vibration, and are perfectly in tune, this line may either be increased or diminished permanently in length according to the difference of phase between the two sets of vibrations. But if the forks be not quite in tune then the length of the image will be found to fluctuate between a maximum and a minimum, thus making the beats sensible to the eye. The vibro­graph is also well suited for the same purpose, and so in an especial manner is Helmholtz’s double siren, in which, by continually turning round the upper box, a note is produced by it more or less out of tune with the note formed by the lower chest, according as the handle is moved more or less rapidly, and most audible beats ensue. We have already explained how beats are used on Scheibler’s tonometer to give a series of forks of known frequencies. Beats also afford an excellent practical guide in the tuning of instruments, but more so for the higher notes of the register, inasmuch as the same number of beats are given by a smaller deviation from unison by two notes of high pitch than by two notes of low pitch. Thus, two low notes of 32 and 30 vibrations respectively, whose interval is therefore or 16/15,i.e. a semitone, give two beats per second, while the same number of beats are given by notes of 32X16 (four octaves higher than the first of the preceding) or 512, and 514 vibrations, which are only slightly out of tune.

*Beats and Dissonance.*—As the interval between two tones, and consequently the number of beats, increases the effect on the ear becomes more and more unpleasant. The sound is jarring and harsh, and we term it a “ dissonance ” or “ discord.” In the middle notes of the musical register the maximum harshness occurs when the beats are about 30. Thus the interval *b'c"* with frequencies 495 and 528, giving 33 beats in a second, is very dissonant. But the interval *b'*♭*c''* gives nearly twice as many beats and is not nearly so dissonant. The minor third *a'c"* with 88 beats per second shows scarcely any roughness, and when the beats rise to 132 per second the result is no longer unpleasant.

We are then led to conclude that beats are the physical founda- tion for dissonance. The frequency of beats giving maximum dissonance rises as we rise higher in the musical scale, and falls as we descend. Thus *b"c'"* and *b'♭c"* have each 66 beats per second, yet the former is more dissonant than the latter. Again *b'c,"* and CG have each 33 beats per second, yet the latter interval is practi­cally smooth and consonant. This beat theory of dissonance was first put forward by Joseph Sauveur (1653-1716) in 1700. Robert Smith (*Harmonies,* 2nd ed., 1759, p. 95) states that Sauveur “ in­ferred that octaves and other simple concords, whose vibrations coincide very often, are agreeable and pleasant because their beats are too quick to be distinguished, be the pitch of the sounds ever so low; and on the contrary, that the more complex consonances whose vibrations coincide seldom are disagreeable because we can distinguish their slow beats; which displease the ear, says he, by reason of the inequality of the sound. And in pursuing this thought he found that those consonances which beat faster than six times in a second are the very same that musicians treat as concords; and that others which beat slower are the discords; and he adds that when a consonance is a discord at a low pitch and a concord at a high one, it beats sensibly at the former pitch but not at the latter.” But Sauveur fixed the limiting number of beats for the discord far too low, and again he gave no account of dissonances such as the seventh, where the frequency of the beats between the fundamentals is far beyond the number which is unpleasant. Smith, though recognizing the unpleasantness of beats, could not accept Sauveur’s theory, and, indeed, it received no acceptance till it was rediscovered by Helmholtz, to whose investigations, recorded in his *Sensations of Tone,* we owe its satisfactory establishment.

Suppose that we start with two simple tones in unison; there is perfect consonance. If one is gradually raised in pitch beating begins, at first easily countable. But as the pitch of the one rises the beats become a jar too frequent to count, and only perhaps to a trained ear recognizable as beats. The two tones are now dissonant, and, as we have seen, about the middle of the scale the maximum dissonance is when there are between 30 and 40 beats per second. If the pitch is raised still further the dissonance lessens, and when there are about 130 beats per second the interval is consonant. If all tones were pure, dissonance at this part of the scale would not occur if the interval were more than a third. But we have to remember that with strings, pipes and instruments gener­ally the fundamental tone is accompanied by overtones, called also “ upper partials,” and beating within the dissonance range may occur between these overtones.

Thus, suppose a fundamental 256 has present with it overtone harmonics 512, 768, 1024, 1280, &c., and that we sound with it the major seventh with fundamental 480, and having harmonics 960, 1440, &c. The two sets may be arranged thus

*c* 256 512 768 1024 1280

*' b* 480 900 1440,

and we see that the fundamental of the second will beat 32 times per second with the first overtone of the first, giving dissonance. The first overtone of the second will beat 64 times per second with the third of the first, and at such height in the scale this frequency will be unpleasant. The very marked dissonance of the major seventh is thus explained. We can see, too, at once how the octave is such a smooth consonance. Let the two tones with their harmonic overtones be

256 512 768 1024 1280 1536

512 1024 1536.

The fundamental and overtones of the second all coincide with overtones of the first.

Take as a further example the fifth with harmonic overtones as under

256 512 768 1024 1280 1536

384 768 1152 1536∙