N (E. C. Pickering, generalized by T. N.

(3) «=Α-(5+μ)ι+α τhicle).

(4) "=A—(Ritz).

(5) «=A-(^+^g/ä-)a (Hicks).

In all cases s represents the succession of integer numbers. In the last case we must put for *r* either s or s+½ according to the nature of the series, as will be explained further on. The first of the forms which contains three disposable constants did good service in the hands of their authors, but breaks down in important cases when odd powers of s have to be introduced in addition to the even powers. The second form contains two or three constants according as N is taken to have the same value for all elements or not. Rydberg favours the former view, but he does not attempt to obtain any very close approximation between the observed and calculated values of the fre- quencies. Equation (3), which E. C. Pickering@@1 used in a special case, presently to be referred to, was put into a more general form by Thiele,@@2 who, however, assumes N to have the same value for all spectra, and not obtaining sufficient agreement, rejects the formula. J. Halm@@3 subsequently showed that if N may differ in different cases, the equation is a considerable improvement on Rydberg’s. It then possesses four adjustable constants, and more can therefore be expected from it. All these forms are put into the shade by that which was introduced by Ritz, led thereto apparently by theoretical considerations. As he takes N to be strictly the same for all elements the equation has only three disposable constants A, *a* and *b.* It is found to be very markedly superior to the other equations. Its chief advantage appears, however, when the relationship between different series of the same element is taken into account. We therefore turn our attention to this relationship.

In the case of those elements in which we can represent the spectrum most completely by a number of series, it is generally found that they occur in groups of three which are closely related to each other. They were called by H. Kayser and F. Paschen “ Haupt serie,” “ 1st Nebenserie,” “ 2nd Nebenserie,” which is commonly translated “ Principal series,” “First subordinate series,” “ Second subordinate series.” These names become inconvenient when, as is generally the case, each of the series splits into groups of two or three, and we have to speak of the second or third number of the first or second subordinate series. Moreover, a false impression is conveyed by the nomenclature, as the second subordinate series is much more closely related to the principal series than the first subordinate series. The present writer, therefore, in his *Theory of Optics,* adopted different names, and called the series respectively the “ Trunk,” the “ Main Branch ” and the “ Side Branch,” the main branch being identical with the second subordinate series; the limit of frequency for high values of *s* is called the “ root ” of the series, and it is found in all cases that the two branches have a common root at some point in the trunk. According to an important law discovered by Rydberg and shortly afterwards independently by the writer, the frequency of the common root of the two branches is obtained by subtracting the frequency of the root of the trunk from that of its least refrangible and strongest member. In the spectra of the alkali metals each line of the trunk is a doublet, and we may speak of a twin trunk springing out of the same root. In the same spectra the lines belonging to the two branches are also doublets. According to the above law the least refrangible member of the trunk being double, there must be two roots for the branches, and this is found to bé the case. In fact the lines of each branch are also doublets, with common difference of frequency. There are, therefore, two main branches and two side branches, but these arc not twins springing out of the same root, but parallel branches springing out of different though closely adjacent roots. It will also be noticed that the least refrangible of the doublets of the

branches must according to the above law correspond to the most refrangible of the doublets of the trunk, and if the components of the doublets have different intensities the stronger components must lie on different sides in the trunk and branch series. This is confirmed by observation. Rydberg discovered a second relationship, which, however, involving the assumed equation connecting the different lines, cannot be tested directly as long as these equations are only approximate. On the other hand the law, once shown to hold approximately, may be used to test the sufficiency of a particular form of equation. These forms all agree in making the frequency negative when s falls below a certain value *sρ.* Rydberg’s second law states that if the main branch series is taken, the numerical value of *np*-1 corresponding to *sp-1* is equal to the frequency of the least refrangible member of the trunk series.

The two laws are best understood by putting the equations in the form given them by Rydberg.

For the trunk series write

*n,* I 1

(5÷m2),

and for the main branch series

*ηia* 1 I

N" (1+m)2 (5÷σ)2'

Here *μf σ* and N are constants, while 5 as before is an integer number.

The difference between the frequencies of the roots (s = ∞) is given by

w"0 ~η'x =N [(1 +σ)2^(1 +μ)2l =n'·

This is the first law.

If further in the two equations we put s = 1, we obtain: *n1=* — n11.

This is the second law.

As has already been mentioned, the law is only verified very roughly, if Rydberg’s form of equation is taken as correctly representing the series. The fact that the addition of the term intro­duced by Ritz not only gives a more satisfactory representation of each series, but verifies the above relationship with a much closer degree of approximation, proves that Ritz’s equation forms a marked step in the right direction. According to him, the following equations represent the connexion between the lines of the three related series.

Trunk series: ΑΚ“[«+αι+4/«\*Ρ"[ι·5+αχ+\*'/(ΐ·5)ψ’

Main Branch Series: =

Side Branch Senes: i-⅛^ = [2φ01+⅛∕22p-[i+c+j∕ssp∙

Here 5 stands for an integer number beginning with 2 for the trunk and 3 for the main branch, and *r* represents the succession of numbers 1∙5, 2∙5, 3∙5, *&c.* As Ritz points out, the first two equations appear only to be particular cases of the form

*η* I I

*(r+βγ*

in which 5 and *r* have the form given above. In the trunk series *s* has the particular value 1∙5, and in the main branch series *s* has the particular value 2, but we -should expect a weaker set of lines to exist corresponding to the trunk series with r=2∙5 or corresponding to the main branch series with s=3, and in fact a whole succession of such series. Taking the Trunk and Main Branch Series, we find they depend altogether on the four constants: *a1, b, al, b1*, while N is a universal constant identical with that deduced from the hydrogen series. As an example of the accuracy obtained we give in the following Table the figures for potassium. The lines of the trunk series are double but for the sake of shortness the least refrangible component is here omitted.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Spectrum of Potassium.* | | | | | | | | |
| Trunk Series. | | | Main Branch. | | | Side Branch. | | |
| S | n | Δ | *r* | *n* | A | 5 | n | Δ |
| 2 | 13o36∙8 | -0-24 | ι∙5 | 12980· 7 | o·oo | 5 | I7i99∙5 | o·oo |
| 3 | 24719∙4 | ÷o·oo | 2∙5 | — | — | 6 | 18709-5 | o·oo |
| 4 | 29006· 7 | -j-o∙12 | 3∙5 | 14465∙3 | o·oo | 7 | i96l1∙2 | 4-o∙16 |
| 5 | 31o73∙5 | -0-05 | 4’5 | 17288∙3 | +o∙2o | 8 | 2o188∙o | ÷o∙7o |
| 6 | 32226∙5 | ÷o∙40 | 5∙5 | 18779∙2 | -j-o∙22 |  |  |  |
| 7 | 32939∙4 | -0-05 | 6-5 | 19662· 3 | -∣-o∙22 |  |  |  |
| 8 | 334o8∙7 | —o∙o8 | 7∙5 | 20224∙7 | + l∙io |  |  |  |
| 9 | 33736∙2 | -0-07 |  |  |  |  |  |  |
| 10 | 33971 \*4 | -o∙23 |  |  |  |  |  |  |

*@@@1 Astrophys. Journ.* (1896), 4, p. 369.

@@@2 Ibid, (1897), 6, p. 65.

*@@@, Trans. Ast. Soc. Edinburgh* (1905), 41, p. 551.