“ leucoxene,” frequently occurs in basic igneous rocks as an alteration product of ilmenite and rutile. (L. J. S.)

**SPHENODON,** or Tuatara. *Sphenodon* s. *Hatteria* (called by Gray after Hatter), with one species, S. *punctatum,* is the sole surviving member of the whole group of *Rhynchocephalia (q.υ.* under Reptiles, *Fossil).* It is one of the few reptiles inhabiting New Zealand; formerly common on the main islands, now restricted to some of the small, uninhabited islands in the Bay of Plenty, where these last “ living fossils ” enjoy the protection of the government. The Maoris call it *ruatara, tuatete or tuatara,* the latter meaning “ having spines." This creature represents an almost ideally generalized type of reptile. The total length of large males is more than two feet, but mature females are scarcely half this size. In general appearance they much resemble the *Agamidae,* especially *Uromastix,* or *Physignathus,* with the massive head, the chisel-shaped front teeth, short legs and erectile crest of cutaneous spines on the head and along the mid-line of the trunk and tail, whilst the rest of the dark olive-green skin is granular, with yellowish specks. But the Agamoid resemblance is only skin-deep, and only the tyro can confound them with any group of Lacertilia. At the same time it is probable that *Sphenodon* stands near the ancestral root of the Lacertilia, before these divided into geckos, chameleons, and lizards proper. The development of this animal has been first studied by G. B. Howes, who quotes the literature bearing upon the whole subject. A good account of the hàbits of the tuatara has been given by Newman. They live upon animals, but these are only taken when alive and moving about, *e.g.* fish, worms, insects. Sluggish in their habits, they sleep during the greater part of the day in their self-dug burrows, and are very fond of lying in the water, and they remain below for hours without breathing. Each individual excavates its own hole, a tunnel leading into a roomy chamber, lined with grass and leaves; part of the habitation is shared socially by a family of petrels, which is said to occupy usually the left side, whilst the tuatara itself lives a solitary life. The male croaks or grunts much during the pairing season; the hard-shelled, long-oval eggs, about 28 mm. long, are laid in holes in the sand, about ten in one nest, from November to January or February. They contain nearly ripe embryos in the following August, but they are not hatched until about thirteen months old; in the meantime they seem to undergo a kind of hibernation, their nasal chambers becoming blocked with proliferating epithelium, which is resolved shortly before hatching during the southern summer. In spite of their imposing, rather noble appearance, when, with their heads erect, they calmly look about with their large quiet eyes, they are dull creatures, but they bite furiously.

For life history see A. K. Newman, *Trans. New Zealand Inst.* (1878), x. 222; Von Haast, ibid. (1881), xiv. 276; Reischek, ibid. xiv. 274; A. Dendy, ibid. (1899), xxxi. 245; *Nature,* 59, 340. For development; G. B. Howes and H. H. Swinnerton, *Trans.. Zoot. Soc.* (1900), xv. 1-86, six plates; A. Dendy, *Quart. Journ. Mic. Sci.* (1899), 42, pp. 1-87, ten plates and ibid. pp. 111-153 (parietal eye); H. Schauinsland, *Arch. mikr. Anat.* (1900), 56, pp. 747-867, plates. For anatomy: A. Günther, *Phil. Trans.* (1867), 157, pp. 595-629, plates; A. K. Newman, quoted above; F. J. Knox, *Trans. New Zealand Inst.* (1869) ii. 17-20; G. Osawa, *Arch. mikr. Anat.* (1898), 51, pp. 481-690, and ibid. 52, pp. 268-366. (H. F. G.)

SPHERE (Gr. *σφαίρα,* a ball or globe), in geometry, the solid or surface traced out by the revolution of a semicircle about its diameter; this is essentially Euclid’s definition;@@1 in the modern geometry of surfaces it is defined as the quadric surface passing through the circle at infinity. Every point is equidistant from a fixed point within the surface; this point is the “ centre,” the constant distance the “ radius,” and any line through the centre and intersecting the sphere is a “ diameter.” All sections of the

sphere are necessarily circles; if the cutting plane contains the centre, the section is said to be “ meridional,” the curve of intersection is a “ great circle,” and the solid cut off a “ hemisphere.” If the plane does not contain the centre, the curve of intersection is a “ small circle,” and the solid cut off is a “ segment.” “ Great ” circles may also be defined as circles on a sphere which pass through the extremities of a diameter; they are familiar as the meridians or lines of longitude of geographers; lines of latitude are “ small circles.” The shortest distance between two points on a sphere is the arc of the great circle containing the points. This proposition is the basis of the “ great circle sailing ” of navigators, and the arc of the great circle is called the “ rhumbline ” or “ loxodromic curve.” The determination of the shortest distance between two small circles on a sphere is given in the article Variations, Calculus of. The extremities of the diameter perpendicular to a small circle are called the “ poles ” of that circle, and the distance from the pole to the circle, measured by the arc of the great circle through the pole, is the “ polar distance ” of the small circle. The solid enclosed by a small circle and the radii vectores from the centre of the sphere is a “ spherical sector ” ; and the solid contained between two spherical sectors standing on copolar small circles is a “ spherical cone.” A “ spherical sector ” and “ spherical cone ” may be also regarded as the solids of revolution of a circular sector about one of its bounding radii, and about any other line through the vertex respectively. The solid intercepted between two parallel planes is a “ zone.”

The geometry of the sphere was studied by the Greeks; Euclid, in book xii. of his *Elements,* discusses various properties of the sphere, and in book xiii. he shows how to inscribe the five regular polyhedra within it. But with the sole exception of proving that the volumes of spheres are in the triplicate ratio of their diameters, a theorem probably due to Eudoxus, no mention is made of its mensuration. This subject was investigated by Archimedes, who, by his “ method of exhaustions,” derived the principal results. He showed that the surface of a segment is equal to the area of the circle whose radius equals the distance from the vertex to the base of the segment; that the surface of the entire sphere is equal to the curved surface of the circumscribing cylinder, and to four times the area of a great circle of the sphere ; and that the volume is two- thirds that of the circumscribing cylinder. To Zenodorus (c. 200- 100 B.c.) is due the important problem in maxima and minima that for a given surface the sphere is the solid of maximum volume. Calling the radius *r,* and denoting by π the ratio of the circumference to the diameter of a circle, the volume is 4/3πr3 and the surface 4πr2.

Archimedes gave his results in the treatise IIϵρι τys σφαlpαs *καl τον κυλίνδρου:* he left unfinished the problem of dividing a sphere into segments whose volumes are in a given ratio. A solution by means of the parabola and hyperbola was given by Dionysodorus of Amisus *(c.* 1st century b.c), and a similar problem—to construct a segment equal in volume to a given segment, and in surface to another segment—was solved by the Arabian mathematician and astronomer, Al Kuhi.

In analytical geometry, the equation to the sphere takes the forms x2+y2+z2=α2, and *r=a,* the first applying to rectangular Cartesian co-ordinates, the second to polar, the origin being in both cases at the centre of the sphere. If the centre be (a, *ß, y),* the Cartesian equation becomes (x — α)2 + (y — *β)2* + (z — γ)2 = *az ;* consequently the general equation is x2+y2+z2+2Ax+ 2By+2Cz+D=0, and it is readily shown that the co-ordinates of the centre are (—A, —B, — C), and the radius A2+B2+C2-D. A sphere can therefore be described so as to satisfy four given conditions. Systems of spheres have characters analogous to those of systems of circles. If *r,* rl be the radii of two spheres, *d* the distance between the centres, and *φ* the angle at which they intersect, then d2 = r2+ r12 + *2rr1* cos φ; hence *2rr1* cos φ=d2- *r2-r12.* This function is named the "power ” of the two spheres, and it is important in the investigation of systems of spheres. If the sphere *r1* degenerate to a point, the function *2rr1* cos *φ* has the limit d2~r2; this is the square of the tangent to the sphere from the point, and is named the “ power of the sphere at the point,” or the “ power of the point with respect to the sphere.” Two spheres intersect in a plane, and the equation to a system of spheres which intersect in a common circle is x2 + y2 + z2+2Ax + D = 0, in which A varies from sphere to sphere, and D is constant for all the spheres, the plane *yz* being the plane of intersection, and the axis of *χ* the line of centres. Corresponding to the radical centre of three circles, it may be shown that four spheres have a radical centre, *i.e.* that there exists a point such that the tangents from this point to the four spheres are equal, and that with this point as centre, and the length of the tangent as radius, a sphere may be described which

@@@l The surfaces formed by revolving a circle about any chord also received attention at the hands of the Greeks. According to Heron and Geminus they were discussed under the name *spire* by Perseus *(c.* 200-100 b.c.), their sections were termed *spiral sections,* and arc probably the same as the *hippopede* of Eudoxus. The surface and solid traced by the revolution of the lesser segment of a circle is termed a “ spindle.” An "anchor ring” or “ tore ” results when a circle revolves about an axis in its plane.