hold over the former, and proceeded to enter into treaties defining the spheres of influence.

The following are some of the chief treaties by which such spheres are defined:—

Great Britain and Portugal as to Africa, August 20, 1890, November 14, 1890 and June 11, 1891. Great Britain and France as to Upper Niger, January 20, 1891; November 15, 1893, as to Lake Chad. Great Britain and France as to Siam, January 15, 1896. The two governments engage to one another “ that neither of them will, without the consent of the other in any case or under any pretext, advance their armed forces into the regions, &c.” They also engage not to acquire within this region any special privilege or advantage which shall not be enjoyed in common, or equally open to Great Britain and France or their nationals and dependants. Great Britain and Italy as to Africa, April 15, 1891; May 5, 1894, as to region of the Gulf of Aden. Congo and Portugal, May 25, 1891, as to “ sphères de souveraineté et d’influence ’’in the region of Lunda. Great Britain, Belgium and Congo, May 12, 1894, as to the sphere of influence of the independent Congo State. Great Britain and Germany, July I, 1890 and November 15, 1893, as to East and Central Africa. Great Britain and Russia as to the spheres of influence to the east of Lake Victoria in the region of the Pamirs, March 11, 1895.

As an example of the promises or engagements in such treaties may be quoted that between Great Britain and Portugal of the 20th of August 1890. Portugal engages that the territory of which the limits are defined in article 3 shall not, without the consent of Great Britain, be transferred to any other power. In the treaty between the same powers of the 14th of November 1890 it is stipulated that neither power will make, tender, accept pro- tectorates, or exercise any act of sovereignty, &c. Sometimes a treaty defining spheres of influence declares that such and such territory shall be neutral.

In the treaty of delimitation between France and Germany of the 15th of March 1894, the line of demarcation of the zones of influence of the two states in the region of Lake Chad is drawn, and they agree to exercise no political influence in such spheres. Each of the states agrees (art. 2) to acquire no territory, to conclude no treaties, to accept no rights of sovereignty, or pro- tectorate, and not “ gêner ou de contester l’influence de l’autre Puissance dans la zone qui lui est reservée.”

Being the result of treaties, arrangements as to spheres of influence bind only the parties thereto. As Mr Olney, in his correspondence with Lord Salisbury in regard to Venezuela, remarked: “ Arrangements as to spheres of influence are new departures, which certain great European Powers have found necessary and convenient in the course of their division among themselves of great tracts of the continent of Africa, and which find their sanction solely in their reciprocal obligations ” (United States No. 2, 1896, p. 27).

Some treaties expressly declare that the arrangement" shall not affect the rights of other powers (Stoerck, *Recueil,* xvi. p. 932). No doubt, however, the tendency is for spheres of influence to become protectorates. It may be mentioned that Germany and Holland have concluded a treaty (Dec. 21, 1897) by which the latter agrees to extradite German criminals in spheres of influence. By an agreement of the 12th of May 1894 between Great Britain and the Congo State, the former granted to the latter a lease of territories comprised within the sphere of influence laid down in the Anglo-German agreement of the 1st of July 1890 (19 *Hertslet,* p. 179).

Somewhat akin to the rights of a state in a sphere of influence are those possessed by Germany in the zone surrounding the protectorate of Kiaochow under the treaty of the 6th of March 1898, and the rights obtained under treaties with China that certain provinces shall not be alienated.

Somewhat similar arrangements as to ports of the sea are not unknown. Grotius in his *Mare liberum* says: “ Illud interim fatemur, potuisse inter gentes aliquas convenire, ut capti in maris hac vel ilia parte, hujus aut illius reipublicae judicium subirent, atque ita ad commoditatem distinguendae jurisdictionis in mari fines describi, quod ipsos quidem earn sibi legem ferentes obligat, at alios populos non item; neque locum cujus proprium facit, sed in personas contrahentium jus constituit ” (c. 5).

The best known example of a claim to a sphere of influence, which is not the result of any treaty, is the Monroe doctrine, first broached by President Monroe in 1823. The Romans had their equivalent to the Monroe doctrine; they forbade any Asiatic king entering Europe and conquering any part of it; the breach of this rule was their chief grievance against Mithradates (Montesquieu, *De la Grandeur et de la decadence des romains,* (c. 6).

Claims somewhat similar to those relating to spheres of influ­ence have been put forward as against the whole world, in virtue of the right of continuity or the doctrine of the hinterland. Sometimes it is called the “ doctrine of contiguity,” or “ droit de vicinité, de priorité, de preemption ou d’enclave.” He who occupies a part of a well-defined close or *fundus,* a parcel of land with artificial or natural boundaries, which enables him to control the whole area, may be said to occupy it. He need not be present everywhere, or enter on every part of it: “ Sufficit quamlibet partem ejus fundi introire, dum mente et cogitatione hac sit, uti totum fundum usque ad terminum velit possidere ” *(Dig.* xli. 2,3). In virtue of a supposed analogy to such occupation, it has been said that the occupation of the mouth of a river is constructive occupation of all its basin and tributaries, and that the occupation of part of a territory extends to all the country of which it forms physically a part. A state, having actually occupied the coast, may claim to reserve to itself the right of occupying from time to time territory lying inland (hinterland). In the discussions as to the western boun­dary of Louisiana between the commissions of the United States and Spain, as to Oregon, as to the claims of the Portuguese in East Africa, and as to the boundaries of Venezuela, the question of the extent of the rights of the discoverer and occupier came up. Portugal actually claimed all territory lying between her African possessions. It has been urged that the subsequent settlement within a reasonable time of the mouth of a river, “ particularly if none of its branches had been explored prior to such discovery, gave the right of occupation, and ultimately of sovereignty, to the whole country drained by such river and its several branches.” Another form of the same doctrine is, that the occupier of a part of the sea-coast thereby acquires rights “ extending into the interior of the country to the sources of the rivers emptying within that coast, to all their branches, and the country they cover ” (Twiss, *Laws of Nations in Time of Peace,* p. 170; Twiss, *Oregon Question,* p. 245; Bluntschli, s. 282; Phillimore, *Commen­taries,* p. 236; Westlake, *International Law,* pt. i. p. 128). Lord Salisbury referred to “ the modem doctrine of hinterland with its inevitable contradictions ” (United States, No. 2, 1896, p. 12). Certainly it is inconsistent with the doctrine, more and more received in recent times, that effective possession is necessary to found a title to sovereignty or control. It is akin to the extravagant claims of the early Portuguese and Spanish navi­gators to territory on which they had never set foot or eyes. The doctrine of the hinterland is likely to become less important, now that Africa has been parcelled out.

Authorities.—Twiss, *Laws of Nations in Time of Peace* (1855); Phillimore, *Commentaries on International Law,* s. 236; Salomon, *L'Occupation des territoires sans maître* (1889); *Correspondence as to Delagoa Bay* (Portugal, No. 1, 1875, p. 191) ; *British Counter Case, Venezuela,* No. 2 (1899), p. 135; *Annuaire de Vinstitut de droit international,* ix. 243; x. 173; *Revue de droit international,* xvii. 113; xviii. 433; xix, 371; *Venezuelan Papers,* No. 4 (1896); J. B. Moore, *Digest of International Law* (1906), i. 268. (J. M.)

**SPHERICAL HARMONICS,** in mathematics, certain functions of fundamental importance in the mathematical theories of gravitation, electricity, hydrodynamics, and in other branches of physics. The term “ spherical harmonic ” is due to Lord Kelvin, and is primarily employed to denote either a rational integral homogeneous function of three variables x, y, *ζ,* which satisfies the differential equation

*∂x2 + ∂y2 + ∂z2 1*

known as Laplace’s equation, or a function which satisfies the differential equation, and becomes a rational integral homo­geneous function when multiplied by a power of (x2+y2+z2)½∙