*Phil. Trans. Roy. Soc.,* 1862), and also independently bξ Clebsch (see his paper, “ Ueber die Reflexion an einer Kugelfläche,” Crelle’s *Journal,* vol. lxi., 1863). The general theory of spherical harmonics of unrestricted degree, order and argument has been treated by Hob- son *(Phil. Trans.,* 1896) ; see also a paper by Barnes in the *Quar. Journ. Math.* 39, p. 97. The functions which bear the name of Bessel were first introduced by Fourier in his investigations on the conduction of heat (see his *Théorie analytique de la chaleur,* 1822); they were employed by Bessel in the theory of planetary motion (see the *Abhandlungen* of the Berlin Academy, 1824). The functions which are now known as Bessel’s functions of degree half an odd integer were employed by Poisson in the theory of the conduction of heat in a solid spherical body (see the *Journ. de l'école polyt.,* 1823, cah. 19). The toroidal functions were introduced by C. Neumann *(Theorie der Elektricitäts- und Warmevertheilung in einem Ringe,* Halle, 1864), and independently by Hicks *(Phil. Trans. Roy.* Soc.,1881). The ellipsoidal harmonics were first investigated by Lamé in connection with the stationary motion of heat in an ellipsoidal body (see Lionville’s *Journal,* 1839, pt. iv. The external ellipsoidal harmonics were introduced by Liouville and Heine (see Lionville’s *Journal,* vol. x., and Crelle’s *Journal,* vol. xxix.). The ellipsoidal harmonics have been considered as expressed in Cartesian co-ordinates by Green (see *Collected Works),* by Ferrers (see his treatise), and by W. D. Niven *(Phil. Trans. Roy. Soc.,* 1892). A method of representing ellipsoidal harmonics in a form adapted for actual use in certain physical problems has been developed by G. H. Darwin *(Phil. Trans.,* vol. 197).

The following treatises may be consulted: Heine, *Theorie der Kugelfunctionen* (2nd ed., 1878, vol. i.; 1881, vol. ii.) ; this treatise gives much information as to the history and literature of the subject; Ferrers, *Spherical Harmonics* (Cambridge, 1881); Todhunter, *The Functions of Laplace, Lamé and Bessel* (Cambridge, 1875); Thomson and Tait, *Natural Philosophy* (1879), App. B. Haentzschel, *Reduction der Potentialgleichung auf gewöhnliche Differ­entialgleichungen* (Berlin, 1893); F. Neumann, *Beiträge zur Theorie der Kugelfunctionen* (Leipzig, 1878); C. Neumann, *Theorie der Bessel'schen Functionen* (Leipzig, 1867); *Ueber die nach Kreis-,Kugel- und Cylinder-functionen fortschreitenden Entwickelungen* (Leipzig, 1881); Lommel, *Studien über die Bessel'schen Functionen* (Leipzig, 1868); Mathieu, *Cours de physique mathématique* (Paris, 1873) ; Pockels, *Ueber die partielle Differentialgleichung* ∆w-∣-fe2u=o (Berlin, 1891); Bôcher, *Ueber die Reihenentwickelungen der Potentialtheorie* (Leipzig, 1894); Gray and Mathews, *Treatise on Bessel's Functions;* Dini, *Serie di Fourier e altre rappresentazione . .* . (Pisa, 1880); Graf and Gubler, *Einleitung in die Theorie der Bessel'schen Functionen* (Berne, 1898); Nielsen, *Handbuch der Theorie der Cylinderfunktionen* (Leipzig, 1904) ; Whittaker, A *Course of Modern Analysis (Cambridge,* 1902); H. Weber, *Die partiellen Differentialgleichungen der Physik* (Bremen, 1900) ; W. E. Byerly, *Fourier's Series and Spherical, Cylin­drical and Ellipsoidal Harmonics* (Boston, 1893). (E. W. H.)

SPHEROID (Gr. *σφαιρα-cιδqs,* like a sphere), a solid resem- bling, but not identical with, a sphere in shape. In geometry, the word is confined to the figures generated by an ellipse revolving about a diameter. If the axis of revolution be the major axis of the ellipse, the spheroid is “ prolate ”; if the minor axis, “oblate”; if any other, “universal.’’

If the generating ellipse has for its equation x2∕a2+y2∕b2 = 1, and revolves about the major axis, *i.e.* the axis of x, the volume of the solid generated is 4/37rα02, and its surface is *2π{b2A-(ab∣e)* sin-1e), where *e* denotes the eccentricity, lf the curve revolve about the minor axis, the volume is ∣7rα20, and the surface isπ(2α24- *(b2∣e)* log (1 +e)∕(ι *—e)*}. The figure of the earth is frequently referred to as an oblate spheroid; this, however, is hardly correct, for the geoid has three unequal axes. The Cartesian equation to a spheroid assumes the forms x2∕α24-(y24-z2)∕fc2 = ι, for the prolate, and (x2+z2)∕α2√-y2∕ft2 = ι, for the oblate, the origin being the centre and the co-ordinate axes the axes of the original ellipse, x2∕α2+y2∕⅛2 = ι, and the line perpendicular to the plane containing them.

In physics, the term “ spheroidal state ” is given to the following phenomenon. If drops of a liquid be placed on a highly heated surface, for example, the top of a stove, the liquid forms a number of tremulous globules which continually circulate internally. There is no visible boiling, although the globule diminishes slowly in size. The theory of the experiment is that the liquid is surrounded by an elastic envelope of its vapour which acts, as it were, as a cushion preventing actual contact of the drop with the plate. On the formation of a similar protective cushion of vapour depends the immunity of such experiments as plunging a hand into a bath of molten metal.

SPHEROMETER (Gr. *σφαίρα ,* a sphere, *μέτρον,* a measure), an instrument for the precise measurement of the radius of a sphere or the thickness of a thin plate. The usual form consists of a fine screw moving in a nut carried on the centre of a small three-legged table; the feet forming the vertices of an equilateral triangle (see figure).

The lower end of the screw and those of the table legs are finely tapered and terminate in hemispheres, so that each rests on a point. If the screw has two turns of the thread to the millimetre the head is usually divided into 500 equal parts, so that differences of 0∙001 millimetre may be measured without using a vernier. A lens, however, may be fitted, in order to magnify the scale divisions. A vertical scale fastened to the table indicates the number of whole turns of the screw and serves as an index for reading the divisions on the head. In order to measure the thickness of a plate the instrument is placed on a perfectly level plane surface and the screw turned until the point just touches; the exact instant when it does so is defined by a sudden diminution of resistance succeeded by a considerable increase. The divided head and scale are read ; the screw is raised ; the thin plate slipped under it; and the process is repeated. The difference between the two readings gives the required thickness. A contact- lever, delicate level or electric contact arrangement may be attached to the spherometer in order to indicate the moment of touching more precisely than is possible by the sense of touch. To measure the radius of a sphere—*e.g.* the curvature of a lens—the spherometer is levelled and read, then placed on the sphere, adjusted until the four points exert equal pressure, and read again. The difference gives the thickness of that portion of the sphere cut off by a plane passing through the three feet. Calling this distance *h,* and the distance between the feet *a,* the radius R is given by the formula R = (α2+3½2)∕6A.

**SPHERULITES** (Gr. *σφαίρα,* sphere, *λlθοs,* stone), in petrology small rounded hodies which commonly occur in vitreous igneous rocks. They are often visible in specimens of obsidian, pitch­stone and rhyolite as globules about the size of millet seed or rice grain, with a duller lustre than the surrounding glassy hase of the rock, and when they are examined with a lens they prove to have a radiate fibrous structure. Under the microscope the spherulites are of circular outline and are composed of thin divergent fibres, which are crystalline and react on polarized light. Between crossed nicols a black cross appears in the spheru- lite; its axes are usually perpendicular to one another and parallel to the crossed wires; as the stage is rotated the cross remains steady; between the black arms there are four bright sectors. This shows that the spherulite consists of radiate, doubly refracting fibres which have a straight extinction; the arms of the black cross correspond to those fibres which are extinguished. The aggregate is too fine grained for us to determine directly of what minerals it is composed.

Spherulites are commonest in acid glassy rocks like those above mentioned, but they occur also in basic glasses such as tachylyte. Sometimes they compose the whole mass; more usually they are surrounded by a glassy or felsitic base. When obsidians are devitrified the spherulites are often traceable, though they may he more or less completely recrystallized or silicified. In the centre of a spherulite there may be a crystal *(e.g.* quartz or felspar) or sometimes a cavity. Occasionally spherulites have zones of different colours, and while most frequently spherical they may he polygonal, or irregular in outline. In some New Zealand rhyolites the spherulites send branching “ cervicorn ” processes (like stags’ horns) outwards through the surrounding glass of the rock. The name axiolites is given to long, elliptical or band-like spherulites.

Occasionally spherulites are met with which are half an inch or more in diameter. If the rock be pounded up fragments of these can he picked out by hand and subjected to analysis, and it is found that from their composition they may be regarded as a mixture of quartz and acid felspar. Direct microscopic evidence as to the presence of these minerals is rarely obtainable. Some authors describe spherulites as consisting of felsite or microfelsite, which also is supposed to he a cryptocrystalline quartzofelspathic substance.