may suffer through repeated changes in their state of stress. It appears that in some if not in all materials a limited amount of stress-variation may be repeated time after time without appre­ciable deterioration in the strength of the piece; in the balance-spring of a watch, for instance, tension and compression succeed each other some 150 millions of times in a year, and the spring works for years without apparent injury. In such cases the stresses lie well within the elastic limits. On the other hand, the toughest bar breaks after a small number of bendings to and fro, when these pass the elastic limits, although the stress may have a value greatly short of the normal ultimate strength. A laborious research by A. Wöhler,@@1 extending over twelve years, gave much important information regarding the effects on iron and steel of very numerous repeated alternations of stress from positive to negative, or between a higher and a lower value with­out change of sign. By means of ingeniously-contrived machines he submitted test-pieces to direct pull, alternated with complete or partial relaxation from pull, to repeated bending in one direction and also in opposite directions, and to repeated twisting towards one side and towards opposite sides. the results show that a stress greatly less than the ultimate strength (as tested in the usual way by a single application of load continued to rupture) is sufficient to break a piece if it be often enough re­moved and restored, or even alternated with a less stress of the same kind. In that case, however, the variation of stress being less, the number of repetitions required to produce rupture is greater. In general, the number of repetitions required to produce rupture is increased by reducing the range through which the stress is varied, or by lowering the upper limit of that range. If the greatest stress be chosen small enough, it may be reduced, removed, or even reversed many million times without destroying the piece. Wohler’s results are best shown by quoting a few figures selected from his experiments. The stresses are stated in centners per square zoll;@@2 in the case of bars subjected to bending they refer to the top and bottom sides, which are the most stressed parts of the bar.

I. Iron bar in direct tension:—

Stress. Number of Applications

Max. Min. causing Rupture.

480 0 800

440 0 106,001

400 0 340,853

360 0 480,853

320 0 10,141,645

440 200 2,373,424

440 240 Not broken with 4 millions.

II. Iron bar bent by transverse load:—

Stress. Number of Bendings

Max. Min. causing Rupture.

550 0 169,750

500 0 420,000

450 0 481,950

400 0 1,320,00

350 0 4,035,400

300 0 Not broken with 48 millions.

III. Steel bar bent by transverse load:—

Stress. Number of Bendings

Max. Min. causing Rupture.

900 0 72,450

900 200 81,200

900 300 156,200

900 400 225,300

900 500 764,900—mean of two trials.

900 600 Not broken with 331/2 millions.

IV. Iron bar bent by supporting at one end, the other end being loaded; alternations of stress from pull to push caused by rotating the bar:—

Stress. Number of Rotations

From + to - causing Rupture.

320 56,430

300 99,000

280 183,145

260 479,490

240 909,810

220 30632,588

200 4,917,992

180 19,l86,791

160 Not broken with 1321/4 millions.

From these and other experiments Wöhler concluded that the wrought iron to which the tests refer could probably bear an indefinite number of stress changes between the limits stated (in round numbers) in the following table (the ultimate tensile strength was about 191/2 tons per sq. in.):—

Stress in Tons per Sq. In.

From pull to push +7 to -7

From pull to no stress 13 to 0

From pull to less pull 19 to 101/2

Hence it appears that the actual strength of this material varies in a ratio which may be roughly given as 3 : 2: 1 in the three cases of (*a*) steady pull, (*b*) pull alternating with no stress, very many times repeated, and *(c)* pull alternating with push, very many times repeated. For steel Wöhler obtained results of a generally similar kind. His experiments were repeated by L. Spangenberg, who extended the inquiry to brass, gun-metal and phosphor-bronze.@@3 A considerable amount of light has been thrown on the nature of fatigue in metals by miscroscopic investigations, which will be referred to presently.

*Resilience.—*A useful application of diagrams showing the relation of strain to stress is to determine the amount of work done in straining a piece in any assigned way. The term “ resilience ” is conveniently used to specify the amount of work done when the strain just reaches the corresponding elastic limit. Thus a rod in simple tension or simple compression has a re­silience per unit of volume = *f*2/2E, where *f* is the greatest elastic pull or push. A blow whose energy exceeds the resilience (reckoned for the kind of stress to which the blow gives rise) must in the most favourable case produce a permanent set; in less favourable cases local permanent set will be produced although the energy of the blow is less than the resilience, in consequence of the strain being unequally distributed. In a plastic material a strain exceeding the limit of elasticity absorbs a relatively large amount of energy, and generally increases the resilience for subsequent strains. Fracture under successive blows, as in the testing of rails by placing them as beams on two supports, and allowing a weight to fall in the middle from a given height, results from the accumulated set which is brought about by the energy of each blow exceeding the resilience.

*Internal Stress.—*Professor James Thomson@@4 pointed out that the effect of any externally applied load depends, to a very material extent, on whether there is or is not initial internal stress, or, in other words, whether the loaded piece is initially in what Professor Karl Pearson has called a state of ease. Internal stress existing without the application of force from without the piece must satisfy the condition that its resultant vanishes over any complete cross-section. It may exist in consequence of set caused by previously applied forces (a case of which instances are given below), or in consequence of previous temperature changes, as in cast iron, which is thrown into a state of internal stress by unequally rapid cooling of the mass. Thus in (say) a spherical casting an outside shell solidifies first, and has become partially contracted by cooling by the time the inside has become solid. The inside then contracts, and its contraction is resisted by the shell, which is thereby compressed in a tangential direction, while the metal in the interior is pulled in the direction of the radius. Allusion has already been made to the fact, pointed out by J. Thomson, that the defect of elasticity under small loads which Hodgkinson discovered in cast iron is probably due to initial stress. In plastic metal a nearly complete state of ease is brought about by annealing; even annealed pieces, however, sometimes show, in the first loading, small defects of elasticity, which are probably due to initial stress, as they disappear when the load is reapplied.

*Microscopic Examination.—*Of all recent aids to a knowledge of the structure of metals, of their behaviour under stress, and of the nature of plastic strain, perhaps the most important is microscopic examination. The microscopic study of metals was initiated by II. C. Sorby as early as 1864 (see *Brit. Assoc. Rep.*

*@@@l Die Festigkeits-Versuche mit Eisen und Stahl* (Berlin, 1870), or *Zeilschr. für Bauwesen* (1860-1870); see also *Engineering* (1871), vol. xi. For early experiments by Fairbairn on the same subject, see *Phil. Trans.* (1864).

@@@2 According to Bauschinger the centner per square zoll in which Wöhler gives his results is equivalent to 6∙837 kilos per sq. cm., or 0-0434 ton per sq. in.

*@@@3 Ueber das Verhalten der Metalle bei wiederholten Anstrengungen* (Berlin, 1875). For interesting notices of the fatigue of metals in railway axles, bridge ties, &c., and results of experiments showing reduced plasticity in fatigued metal, see Sir B. Baker’s address to the Mechanical Section of the British Association (1885). In many of the cases where the fatigue of metals occurs in engineering practice the phenomenon is complicated by the occurrence of blows or shocks whose energy is absorbed in producing strains often exceeding the elastic limits, sometimes of a very local character in consequence of the inertia of the strained pieces. Such shocks may cause an accumulation of set which finally leads to rupture in a way that is not to be confused with ordinary fatigue of strength. The effects of the accumulation may be removed by annealing.

*@@@4 Camb, and Dub. Math. Journ.* (Nov. 1848).