of each figure, because they were strictly relative for all angles measured with the same instrument and under similar circumstances and conditions, as was almost always the case for each single figure. But in the final reductions, when numerous chains of triangles composed of figures executed with different instruments and under different circumstances came to be adjusted simultaneously, it was necessary to modify the original weights, on such evidence of thc precision of the angles as might be obtained from other and more reliable sources than the actual measures of the angles. This treatment will now be described.

Values of theoretical error for groups of angles measured with the same instrument and under similar conditions may be obtained in three ways—(i.) from the squares of the reciprocals of the weight *w* deduced as above from the measures of such angle, (ii.) from the magnitudes of the excess of the sum of the angles of each triangle above 180°+ the spherical excess, and (iii.) from the magnitudes of the corrections which it is necessary to apply to the angles of polygonal figures and networks to satisfy the several geometrical conditions.

Every figure, whether a single triangle or a polygonal network, was made consistent by the application of corrections to the observed angles to satisfy its geometrical conditions. The three angles of every triangle having been observed, their sum had to be made=18o° + the spherical excess; in networks it was also necessary that the sum of the angles measured round the horizon at any station should be exactly = 360°, that the sum of the parts of an angle measured at different times should equal the whole and that the ratio of any two sides should be identical, whatever the route through which it was com­puted. These are called the *triangular, central, toto-partial* and *side* conditions; they present *n* geometrical equations, which contain *t* unknown quantities, the errors of the observed angles, *t* being always >n. When these equations are satisfied and the deduced values of errors are applied as corrections to the observed angles, the figure becomes consistent. Primarily the equations were treated by a method of successive approximations; but afterwards they were all solved simultaneously by the so-called method of minimum squares, which leads to the most probable of any system of correc­tions.

The angles having been made geometrically consistent *inter se* in each figure, the side-lengths are computed from the base-line onwards by Legendre's theorem, each angle being dimin­ished by one-third of the spherical excess of the triangle to which it appertains. The theorem is applicable without sensible error to triangles of a much larger size than any that are ever measured.

Λ station of origin being chosen of which the latitude and longitude are known astronomically, and also the azimuth of one of the surrounding stations, the differences of latitude and longitude and the reverse azimuths are calculated in succession, for all the stations of the triangulation, by Puissant’s formulae *(Traité de géodésie,* 3rd ed., Paris, 1842).

*Problem.*—Assuming the earth to be spheroidal, let A and B be two stations on its surface, and let the latitude and longitude of A be known, also the azimuth of B at A, and the distance between A and B at the mean sea-level; we have to find the latitude and longitude of B and the azimuth of A at B.

The following symbols are employed: *a* the major and *b* the ιa5-⅛21 J

—^5—f *; ρ* the radius of

fl(l —∙C2)

curvature to the meridian in latitude λ, = ~~1~~~~1~~~~-~~~~e~~~~⅛∣∏~~gχ~~ι'¾~~'' *v* the normal to the meridian in latitude λ, = ~~1~~~~1~~ ~~-~~~~e~~~~⅛i~~~~n~~~~2χjp~~ λ and *L* the given latitude and longitude of A; λ + ∆λ and *L* + ∆*L* the required lati­tude and longitude of B; *A* the azimuth of B at A; *B* the azimuth of A at B; Δ*A* = *B —* (*π*+*A*); *c* the distance between Aand B. Then, all azimuths being measured from the south, we have

*c . „*

cos *A* cosec 1

P

I *ci*

—— ■—sin’X tan λ coscc 1"

∆λ" = ' *∙, pet fΛ* "(3)

—⅞- — - 5 cos’/l sin 2λ cosec 1"

4 *p∙v* ι — e3

I

÷g^~2 sin’/l cos A(ι+3 tan,λ) coscc 1"

*c* sin *A ,,*

— ∙,τ coscc I

*V* COS Λ

l I c2sin 2√1 tan λ .,

+~ % —ττ cosec I

2 r cos λ

= I ci (1÷3 tan2λ) sin *2A* cos√l ,t (4)

6 v’ COS^λ cθsec 1

l I r3 sin5.4 tan’ λ ,,

+t,^3 —r∑7^t—cosec ι

3 *r* cos λ

--sin *A* tan λ coscc ι"

∆∕l"or ι+2tan4λ+≤^∣sin2ricosec I"

B-(τ-+-4)= -^∣+taπ,λ )l≡Jpsiπ 24 c0a4 coscc i∕∕

I

+g ^jsinM tan λ (1+2 tan5λ) cosec 1"

Each Δ is the sum of four terms symbolized by δ1, δ2, δ3 and δ4;the calculations arc so arranged as to produce these terms in the order *δλ, δL,* and *δA*, each term entering as a factor in calculating the following term. The arrangement is shown below in equations in which the symbols *P, Q, . . . Z* represent the factors which depend on the adopted geodetic constants, and vary with the latitude; the logarithms of their numerical values are tabulated in the *Auxiliary Tables to Facilitate the Calculations of the Indian Survey, δ*1*λ = — P.cosA.c δ1L=*+δ1*λ*.*Q*secλ.tan*A* δ1*A*=+δ1*L*.sinλ]

*δ3X =* +δι∕l. *R.* sin/l.c δjL = — δ3λ. 5.C0t∕l *δsA* ≡ -j-δaL. *F ,,·.*

*δ,λ=-δiA.V .cotA* δsL=+δ3λ. i7.sin∕Lc δ3A = +δsL.IP pb>

δ<λ =—ösA.JV.tanA δ⅛L =-ΡδΑ. F.tan√l δ,∕l ≡ -j-δ⅜L.*Z*

The calculations described so far suffice to make the angles of the several trigonometrical figures consistent *inter se,* and to give preliminary values of the lengths and azimuths of the sides and the latitudes and longitudes of the stations. The results are amply sufficient for the requirements of the topographer and land surveyor, and they are published in preliminary charts, which give full numerical details of latitude, longitude, azimuth and side-length, and of height also, for each portion of the triangulation—secondary as well as principal—as executed year by year. But on the com­pletion of the several chains of triangles further reductions became necessary, to make the triangulation everywhere consistent *inter se* and with the verificatory base-lines, so that the lengths and azimuths of common sides and the latitudes and longitudes of common stations should be identical at the junctions of chains and that the measured and computed lengths of the base-lines should also be identical.

As an illustration of the problem for treatment, suppose a combination of three meridional and two longitudinal chains com­prising seventy-two single triangles with a base-line at each corner as shown in the accompanying diagram (fig. 2); suppose the three angles of every triangle to have been measured and made consistent. Let A be the origin, with its latitude and longitude given, and also the length and azimuth of the adjoining base-line. With these data processes of cal­culation are carried through the triangulation to obtain the lengths and azimuths of the sides and the latitudes and longitudes of the stations, say in the fol­lowing order: from A through B to E, through F to E, through F to D, through F and E to C, and through F and D to C. Then there are two values of side, azimuth, latitude and longitude at E—one from the right-hand chains via B, the other from the left-hand chains via F; similarly there are two sets of values at C; and each of the base-lines at B, C and D has a calculated as well as a measured value. Thus eleven absolute errors are presented for dispersion over the triangulation by the application of the most appropriate correction to each angle, and, as a preliminary to the determination of these corrections, equations must be con­structed between each of the absolute errors and the unknown errors of the angles from which they originated. For this purpose assume *X* to be the angle opposite the flank side of any triangle, and *Y* and *Z* the angles opposite the sides of continuation; also let *x, y* and z be the most probable values of the errors of the angles which will satisfy the given equations of condition. Then each equation may be expressed in the form [*ax+by+cz*] = E, the brackets indicating a summation for all the triangles involved. We have first to ascertain the values of the coefficients *a, b* and *c* of the unknown quantities. They are readily found for the side equations on the circuits and between the base-lines, for *x* does not enter them, but only *y* and z, with coefficients which are the cotangents of *Y* and *Z,* so that these equations are simply [cot *Y.y—*cot.[Z.z] = E. But three out of four of the circuit equations are geodetic, corre­sponding to the closing errors in latitude, longitude and azimuth, and in them the coefficients are very complicated. They are ob­tained as follows. The first term of each of the three expressions for ∆λ, ∆*L*, and *B* is differentiated in terms of *c* and *A,* giving

d.∆λ = ∆λ ∣ <L4 tan A sini"j

*d.ΔL≈ ∆L^+dA* cot *A* sin 1" ∣ - (7)

*dB =dA+∆A* j *-j+dA* cot *A* sin 1"