to form a table of prime numbers the process is theoretically simple and rapid, for we have only to range all the numbers in a line and strike out every second number beginning from 2, every third beginning from 3, and so on, those that remain being primes. Even when the tabular results are constructed separately, the method of differences or other methods con­necting together different tabular results may afford valuable verifications. By having recourse to tables not only does the computer save time and labour, but he also obtains the certainty of accuracy.

The invention of logarithms in 1614, followed immediately by the calculation of logarithmic tables, revolutionized all the methods of calculation; and the original work performed by Henry Briggs and Adrian Vlacq in calculating logarithms in the early part of the 17th century has in effect formed a portion of every arithmetical operation that has since been carried out by means of logarithms. And not only has an incredible amount of labour been saved,@@1 but a vast number of calculations and researches have been rendered practicable which otherwise would have been beyond human reach. The mathematical process that underlies the tabular method of obtaining a result may be indirect and complicated; for example, the logarithmic method would be quite unsuitable for the multiplication of two numbers if the logarithms had to be calculated specially for the purpose and were not already tabulated for use. The arrange­ment of a table on the page and all typographical details—such as the shape of the figures, their spacing, the thickness and placing of the rules, the colour and quality of the paper, &c.— are of the highest importance, as the computer has to spend hours with his eyes fixed upon the book; and the efforts of eye and brain required in finding the right numbers amidst a mass of figures on a page and in taking them out accurately, when the computer is tired as well as when he is fresh, are far more trying than the mechanical action of simple reading. Moreover, the trouble required by the computer to *learn* the use of a table need scarcely be considered; the important matter is the time and labour saved by it after he has learned its use.

In the following descriptions of tables an attempt is made to give an account of all those that a computer of the present day is likely to use in carrying out arithmetical calculations. Tables relating to ordinary arithmetical operations are first described, and afterwards an account is given of the most useful and least technical of the more strictly mathematical tables, such as factorials, gamma functions, integrals, Bessel’s func­tions, &c. Nearly all modern tables are stereotyped, and in giving their titles the accompanying date is either that of the original stereotyping or of the *tirage* in question. In tables that have passed through many editions the date given is that of the edition described. A much fuller account of general tables published previously to 1872, by the present writer, is contained in the British Association *Report* for 1873, pp. 1-175.

*Tables of Divisors (Factor Tables) and Tables of Primes.—*The existing factor tables extend to 10,000,000. In 1811 L. Chernac published at Deventer his *Cribrum arithmelicum,* which gives all the prime divisors of every number not divisible by 2, 3, or 5 up to 1,020,000. In 1814-1817 J. C. Burckhardt published at Paris his *Tables des diviseurs,* giving the least divisor of every number not divisible by 2, 3, or 5 up to 3,036,000. The second million was issued in 1814, the third in 1816, and the first in 1817. The corresponding tables for the seventh, eighth, and ninth millions were calculated by Z. Dase and issued at Hamburg in 1862, 1863, and 1865. Dase died suddenly in 1861 during the progress of the work, and it was completed by H. Rosenberg. Dase's calcula­tion was performed at the instigation of Gauss, and he began at 6,000,000 because the Berlin Academy was in possession of a manu­script presented by Crelle extending Burckhardt’s tables from 3,000,000 to 6,000,000. This manuscript was found on examina­tion to be so inaccurate that the publication was not desirable, and accordingly the three intervening millions were calculated and published by James Glaisher, the *Factor Table for the Fourth*

*Million* appearing at London in 1879, and those for the fifth and sixth millions in 1880 and 1883 respectively (all three millions stereotyped). The tenth million, though calculated by Dase and Rosenberg, has not been published. The nine quarto volumes *(Tables des diviseurs,* Paris, 1814-1817; *Factor Tables,* London, 1879-1883; *Factoren-Tafeln,* Hamburg, 1862-1865) thus form one uniform table, giving the least divisor of every number not divisible by 2, 3, or 5, from unity to nine millions. The arrange­ment of the results on the page, which is due to Burckhardt, is admirable for its clearness and condensation, the least factors for 9000 numbers being given on each page. The tabular portion of each million occupies 112 pages. The first three millions were issued separately, and also bound in one volume, but the other six millions are all separate. Burckhardt began the publication of his tables with the second million instead of the first, as Chernac’s factor table for the first million was already in existence. Burck­hardt’s first million does not supersede Chernac’s, as the latter gives all the prime divisors of numbers not divisible by 2, 3, or 5 up to

1, 020,000. It occupies 1020 pages, and Burckhardt found it very accurate; he detected only thirty-eight errors, of which nine were due to the author, the remaining twenty-nine having been caused by the slipping of type in the printing. The errata thus discovered are given in Burckhardt’s first million. Other errata are contained in Allan Cunningham's paper referred to below.

Burckhardt gives but a very brief account of the method by which he constructed his table; and the introduction to Dase's millions merely consists of Gauss’s letter suggesting their con­struction. The Introduction to the *Fourth Million* (pp. 52) con­tains a full account of the method of construction and a history of factor tables, with a bibliography of writings on the subject. The Introduction (pp. 103) to the *Sixth Million* contains an enumera­tion of primes and a great number of tables relating to the dis­tribution of primes in the whole nine millions, portions of which had been published in the *Cambridge Philosophical Proceedings* and elsewhere. A complete list of errors in the nine millions was published by J. P. Gram *(Acta mathematica,* 1893, 17, p. 310). These errors, 141 in number, and which affect principally the second, third, eighth, and ninth millions, should be carefully cor­rected in all the tables. In 1909 the Carnegie Institution of Wash­ington published a factor table by Prof. D. N. Lehmer which gives the least factor of all numbers not divisible by 2, 3, 5, or 7, up to ten millions. This table, which covers a range of 21,000 numbers on a single page, was reproduced by photography from a type­written copy of the author’s original manuscript. The introduction contains a list of errata in the nine millions previously published, completely confirming Gram's list.

The factor tables which have just been described greatly exceed both in extent and accuracy any others of the same kind, the largest of which only reaches 408,000. This is the limit of Anton Felkel's *Tafel aller einfachen Factoren* (Vienna, 1776), a remark­able and extremely rare book@@2 nearly all the copies having been destroyed. Georg Vega *(Tabulae,* 1797) gave a table showing all the divisors of numbers not divisible by 2, 3, or 5 up to 102,000, followed by a list of primes from 102,000 to 400,313. In the earlier editions of this work there are several errors in the list, but these are no doubt corrected in J. A. Hülsse's edition (1840).

J. Salomon (Vienna, 1827) gives the least divisor of all numbers not divisible by 2, 3, or 5, up to 102,011, and B. Goldberg *(Primzahlen und Factoren-Tafeln,* Leipzig, 1862) gives all factors of numbers not divisible by 2, 3, or 5 up to 251,650. H. G. Köhler *(Logarithmisch-trigonometrisches Handbuch,* 1848 and subsequent editions) gives all factors of numbers not prime or divisible by 2, 3, 5, or 11 up to 21,525. Peter Barlow (*Tables*, 1814) and F. Schaller *(Primzahlen-Tafel,* Weimar, 1855) give all factors of *all* numbers up to 10,000. Barlow’s work also contains a list of primes up to 100,103. Both the factor table and the list of primes are omitted in the stereotyped (1840) reprint. Full lists of errata in Chernac (1811), Barlow (1814), Hülsse’s Vega (1840), Köhler (1848), Schaller (1855), and Goldberg (1862) are contained in a paper by Allan Cunningham *(Mess. of Math.,* 1904, 34, p. 24; 1905, 35, p. 24). V. A. Le Besgue *(Tables diverses pour la décom­position des nombres,* Paris, 1864) gives in a table of twenty pages, the least factor of numbers not divisible by 2, 3, or 5 up to 115,500. In Rees’s *Cyclopaedia* (1819), article “ Prime Numbers,” there is a list of primes to 217,219 arranged in decades. The *Fourth Million* (1879) contains a list of primes up to 30,341. The fourth edition of the *Logarithmic Tables* (London, and Ithaca, N.Y., 1893) of G. W. Jones of Cornell University contains a table of ali the factors of numbers not divisible by 2 or 5 up to 20,000. In the case of primes the ten-place logarithm is given. This table does not occur in the third edition (Ithaca, N.Y., 1891). On the first page of the *Second Million* Burckhardt gives the first nine multiples of the primes to 1423; and a smaller table of the same kind, extending only to 313, occurs in Lambert’s *Supplementa* (1798). Several papers contain lists of high primes *(i.e.* beyond the range of the

@@@1 Referring to factor tables, J. H. Lambert wrote *(Supplementa tabularum,* 1798, p. xv.): “ Universalis finis talium tabularum est ut semel pro semper computetur quod saepius de novo compu- tandum foret, et ut pro omni casu computetur quod in futurum pro quovis casu computatum desiderabitur.” This applies to all tables.

@@@2 For information about it, see a paper on “ Factor Tables,” in *Camb. Phil. Proc.* (1878), iii. 99-138, or the Introduction to the *Fourth Million.*