factor tables). Among these may be mentioned two, by Allan Cunningham and H. J. Woodall jointly, in the *Mess. of Math.,* 1902, 31, p. 165; 1905, 34, p. 72. See also the papers on factoriza­tions of high numbers referred to under *Tables relating to the Theory of Numbers.* The Vienna Academy possesses the manuscript of an immense factor table extending to 100,000,000, constructed many years ago by J. P. Kulik (1793-1863) (see *Ency. math.* MTss., 1900- 1904, i. 952, and Lehmer’s *Factor Table,* p. ix.).

*Multiplication Tables.—*A multiplication table is usually of double entry, the two arguments being the two factors; when so arranged it is frequently called a Pythagorean table. The largest and most useful work is. A. L. Crelle’s *Rechentafeln* (Bremiker's edition, 1857, stereotyped; many subsequent editions with German, French, and English title-pages), which gives in one volume all the products up to 1000x1000, so arranged that all the multiples of any one number appear on the same page. The original edition was pub­lished in 1820 and consisted of two thick octavo volumes. The second (stereotyped) edition is a convenient folio volume of 450 pages.@@1 In 1908 an entirely new edition, edited by O. Seeliger, was published in which the multiples of 10, 20, ..., 990 (omitted in previous editions) are included. This adds 50 pages to the volume, but removes what has been a great drawback to the use of the tables. Other improvements are that the tables are divided off horizontally and vertically by lines and spaces, and that, for calculations in which the last two figures are rejected, a mark has been placed to show when the last figure retained should be increased. Two other tables of the same extent (1000×1000), but more condensed in arrangement, are H. C. Schmidt’s *Zahlenbuch* (Aschersleben, 1896), and A. Henselin's *Rechentafel* (Berlin, 1897). An anonymous table, published at Oldenburg in i860, gives products up to 500×509, and Μ. Cordier, *Le Multiplicateur de trois cents carrés* (Paris, 1872), gives a multiplication table to 300×300 (intended for commercial use). In both these works the product is printed in full. The four following tables are for the multiplica­tion of a number by a single digit. (1) A. L. Crelle, *Erleichterungs­tafel für jeden, der zu rechnen hat* (Berlin, 1836), a work extending to 1000 pages, gives the product of a number of seven figures by a single digit, by means of a double operation of entry. Each page is divided into two tables: for example, to multiply 9382477 by 7 we turn to page 825, and enter the right-hand table at line 77, column 7, where we find 77339; we then enter the left-hand table on the same page at line 93, column 7, and find 656, so that the product required is 65677339. (2) C. A. Bretschneider, *Pro­*

*duktentafel* (Hamburg and Gotha, 1841), is somewhat similar to Crelle’s table, but smaller, the number of figures in the multi­plicand being five instead of seven. (3) In S. L. Laundy, *A Table of Products* (London, 1865), the product of any five-figure number by a single digit is given by a double arrangement. The extent of the table is the same as that of Bretschneider’s, as also is the principle, but the arrangement is different, Laundy's table occupy­ing only 10 pages and Bretschneider’s 99 pages. (4) G. Diakow’s *Multiplikations-Tabelle* (St Petersburg, 1897) is of the same extent as Bretschneider’s table but occupies 1000 pages. Among tables extending to 100 x 1000 *(i.e.* giving the products of two figures by three) may be mentioned C. A. Müller's *Multiplications-Tabellen* (Karlsruhe, 1891). The tables of L. Zimmermann *(Rechentafeln,* Liebenwerda, 1896) and J. Riem *(Rechentabellen für Multiplica­tion,* Basel, 1897) extend to 100 x 10,000. In a folio volume of 500 pages J. Peters *(Rechentafeln für Multiplikation und Division mit ein- bis vierstelligen Zahlen,* Berlin, 1909) gives products of four figures by two. The entry is by the last three figures of the multi­plicand, and there are 2000 products on each page. Among earlier tables, the interest of which is mainly historical, mention may be made of C. Hutton’s *Table of Products and Powers of Numbers* (London, 1781), which contains a table up to 100 × 1000, and J. P. Gruson’s *Grosses Einmaleins von Eins bis Hunderttausend* (Berlin, 1799)—a table of products up to 9 x 10,000. The author’s intention was to extend it to 100,000, but only the first part was published. In this book there is no condensation or double arrangement; the pages are very large, each containing 125 lines.

*Quarter-Squares.—*Multiplication may be performed by means of a table of single entry in the manner indicated by the formula—

*' ab=Ma+bV-Ha-bV.*

Thus with a table of quarter-squares we can multiply together any two numbers by subtracting the quarter-square of their difference from the quarter-square of their sum. The largest table of quarter-squares is J. Blater’s *Table of Quarter-Squares of all whole numbers from I to 200,000* (London, 1888),@@2 which gives quarter-squares of every number up to 200,000 and thus yields directly the product of any two five-figure numbers. This fine table is well printed and arranged. Previous to its publication the largest table was S. L. Laundy’s *Table of Quarter-Squares of all numbers up to* 100,000 (London, 1856), which is of only half the extent, and therefore is only directly available when the sum of the two numbers to be multiplied does not exceed 100,000.

Smaller works are J. I. Centnerschwer, *Neuerfundene Multiplica­tions- und Quadrat-Tafeln* (Berlin, 1825), which extends to 20,000, and J. Μ. Merpaut, *Tables arithmonomiques* (Vannes, 1832), which extends to 40,000. In Merpaut’s work the quarter-square is termed the “ arithmone.” L. J. Ludolf, who published in 1690 a table of squares to 100,000 (see next paragraph), explains in his introduction how his table may be used to effect multiplications by means of the above formula; but the earliest book on quarter­squares is A. Voisin, *Tables des multiplications, ou logarithmes des nombres entiers depuis* 1 *jusqu’à* 20,000 (Paris,' 1817). By a logarithm Voisin means a quarter-square, *i.e.* he calls *a* a root and ¼a2 its logarithm. On the subject of quarter-squares, &c·, see *Phil. Mag.* [v.] 6, p. 331.

*Squares, Cubes, &c., and Square Roots and Cube Roots.—*The most convenient table for general use is P. Barlow's *Tables* (Useful Know­ledge Society, London, from the stereotyped plates of 1840), which gives squares, cubes, square roots, cube roots, and reciprocals to 10,000. These tables also occur in the original edition of 1814. The largest table of squares and cubes is J. P. Kulik, *Tafeln der Quadrat- und Kubik-Zahlen* (Leipzig, 1848), which gives both as far as 100,000. Blater’s table of quarter-squares already mentioned gives squares of numbers up to 100,000 by dividing the number by 2; and up to 200,000 by multiplying the tabular result by 4. Two early tables give squares as far as 100,000, viz. Maginus, *Tabula tetragonica* (Venice, 1592), and Ludolf, *Tetragonometria tabularia* (Amsterdam, 1690); G. A. Jahn, *Tafel der Quadratund Kubikwurzeln* (Leipzig, 1839), gives squares to 27,000, cubes to 24,000, and square and cube roots to 25,500, at first to fourteen decimals and above 1010 to five. E. Gélin *(Recueil de tables numériques,* Huy, 1894) gives square roots (to 15 places) and cube roots (to 10 places) of numbers up to 100. C. Hutton, *Tables of Products and Powers of Numbers* (London, 1781), gives squares up to 25,400, cubes to 10,000, and the first ten powers of the first hundred numbers. P. Barlow, *Mathematical Tables* (original edition, 1814), gives the first ten powers of the first hundred numbers. The first nine or ten powers arc given in Vega, *Tabulae* (1797), and in Hülsse’s edition of the same (1840), in Köhler, *Hand­buch* (1848), and in other collections. C. F. Faà de Bruno, *Calcul des erreurs* (Paris, 1869), and J. H. T. Müller, *Vierstellige Loga­rithmen* (1844), give squares for use in connexion with the method of least squares. Four-place tables of squares are frequently given in five- and four-figure collections of tables. Small tables often occur in books intended for engineers and practical men. S. Μ. Drach *(Messenger of Math.,* 1878, 7, p. 87) has given to 33 places the cube roots (and the cube roots of the squares) of primes up to 127. Small tables of powers of 2, 3, 5, 7 occur in various collections. In Vega’s *Tabulae* (1797, and the subsequent editions, including Hülsse’s) the powers of 2, 3, 5 as far as the 45th, 36th, and 27th respectively are given; they also occur in Köhler’s *Hand­buch* (1848). The first 25 powers of 2, 3, 5, 7 are given in Salomon, *Logarithmische Tafeln* (1827). W. Shanks, *Rectification of the Circle* (1853), gives every 12th power of 2 up to 2721. A very valuable paper (“ Power-tables, Errata ”) published by Allan Cunningham in the Messenger *of Math.,* 1906, 35, p. 13, contains the results of a careful examination of 27 tables containing powers higher than the cube, with lists of errata found in each. Before using any power table this list should be consulted, not only in order to correct the errata, but for the sake of references and general information in regard to such tables. In an appendix (p. 23) Cunningham gives errata in the tables of squares and cubes of Barlow (1814). Jahn (1839), and Kulik (1848).

*Triangular Numbers.—*E. de Joncourt, *De natura et praeclaro usu simplicissimae speciei numerorum trigonalium* (The Hague, 1762), contains a table of triangular numbers up to 20,000: viz. ½n(n+1) is given for all numbers from n = 1 to 20,000. The table occupies 224 pages.

*Reciprocals.—*P. Barlow’s *Tables* (1814 and 1840) give reciprocals up to 10,000 to 9 or 10 places; and a table of ten times this extent is given by VV. H. Oakes, *Table of the Reciprocals of Numbers from I to 100,000* (London, 1865). This table gives seven figures of the reciprocal, and is arranged like a table of seven-figure logarithms, differences being added at the side of the page. The reciprocal

@@@1 Only one other multiplication table of the same extent as Crelle’s had appeared previously, viz. Herwart von Hohenburg’s *Tabulae arithmeticae τrpoσOaφaιpισcωs universales* (Munich, 1610), a huge folio volume of more than a thousand pages. It appears from a correspondence between Kepler and von Hohenburg, which took place at the end of 1608, that the latter used his table when in manuscript for the performance of multiplications in general, and that the occurrence of the word *prosthaphaeresis* on the title is due to Kepler, who pointed out that by means of the table spherical triangles could be solved more easily than by Wittich’s prosthaphaeresis. The invention of logarithms four years later afforded another means of performing multiplications, and von Hohenburg’s work never became generally known. On the method of prosthaphaeresis, see Napier, John, and on von Hohenburg’s table, see a paper “ On multiplication by a Table of Single Entry,” *Phil. Mag.,* 1878, ser. v., 6, p. 331.

@@@2 The actual place of publication (with a German title, &c.) is Vienna. The copies with an English title, &c., were issued by Trübner; and those with a French title, &c., by Gauthier-Villars. All bear the date 1888.