of a number of five figures is therefore taken out at once, and two more figures may be interpolated for as in logarithms. - R. Picarte, *La Division réduite à une addition* (Paris, 1861), gives to ten signi­ficant figures the reciprocals of the numbers from 10,000 to ιoo,ooo, and also the first nine multiples of these reciprocals. J. C. Houzcau gives the reciprocals of numbers up to too to 20 places and their first nine multiples to 12 places in the *Bulletin of the Brussels Academy,* 1875, 40, p. 107. E. Gélin *(Recueil de tables numériques,* Huy, 1894) gives reciprocals of numbers to 1000 to 10 places.

*Tables for the Expression of Vulgar Fractions as Decimals.—* Tables of this kind have been given by Wucherer, Goodwyn and Gauss. W. F. Wucherer, *Beyträge zum allgemeinem Gebrauch der Decimalbrüche* (Carlsruhe, 1796), gives the decimal fractions (to 5 places) for all vulgar fractions whose numerator and denominator are each less than 50 and prime to one another, arranged according to denominators. The most extensive and elaborate tables that have been published are contained in Henry Goodwyn’s *First Centenary of Tables of all Decimal Quotients* (London, 1816), *A Tabular Series of Decimal Quotients* (1823), and *A Table of the Circles arising from the Division of a Unit or any other Whole Number by all the Integers from 1 to 1024* (1823). The *Tabular Series* (1823), which occupies 153 pages, gives to 8 places the decimal corresponding to every vulgar fraction less than 99/991 whose numerator and denomi­nator do not surpass 1000. The arguments are not arranged according to their numerators or denominators, but according to their magnitude, so that the tabular results exhibit a steady increase from .001 ( = 1/1000) to ∙09989909 ( = 99/991). The author intended the table to include all fractions whose numerator and denominator were each less than 1000, but no more was ever published. The *Table of Circles* (1823) gives all the periods of the circulating decimals that can arise from the division of any integer by another integer less than 1024. Thus for 13 we find ·076923 and ∙153846, which are the only periods in which a fraction whose denominator is 13 can circulate. The table occupies 107 pages, some of the periods being of course very long *(e.g.,* for 1021 the period contains 1020 figures). The *First Centenary* (1816) gives the complete periods of the reciprocals of the numbers from 1 to 100. Goodwyn’s tables are very scarce, but as they are nearly unique of their kind they deserve special notice. A second edition of the *First Centenary* was issued in 1818 with the addition of some of the *Tabular Series,* the numerator not exceeding 50 and the denominator not exceeding 100. A posthumous table of C. F. Gauss’s, entitled "Tafel zur Verwandlung gemeiner Brüche mit Nennern aus dem ersten Tausend in Decimalbrüche,” occurs in vol. ii. pp. 412-434 of his *Gesammelte Werke* (Göttingen, 1863), and resembles Goodwyn’s *Table of Circles.* On this subject see a paper “On Circulating Decimals, with special reference to Henry Goodwyn’s *Table of Circles* and *Tabular Series of Decimal Quotients,”* in *Camb. Phil. Proc.,* 1878, 3, p. 185, where is also given a table of the numbers of digits in the periods of fractions corresponding to denominators prime to 10 from 1 to 1024 obtained by counting from Goodwyn’s table. See also under *Circulating Decimals* (below).

*Sexagesimal and Sexcentenary Tables.*—Originally all calculations were sexagesimal ; and the relics of the system still exist in the division of the degree into 60 minutes and the minute into 60 seconds. To facilitate interpolation, therefore, in trigonometrical and other tables the following large sexagesimal tables were con­structed. John Bernoulli, *A Sexcentenary Table* (London, 1779), gives at once the fourth term of any proportion of which the first term is 600" and each of the other two is less than 600\*; the table is of double entry, and may be described as giving the value of xy/600 correct to tenths of a second, *x* and *y* each containing a number of seconds less than 600. Michael Taylor, *A Sexagesimal Table* (London, 1780), exhibits at sight the fourth term of any proportion where the first term is 60 minutes, the second any number of minutes less than 60, and the third any number of minutes and seconds under 60 minutes; there is also another table in which the third term is any absolute number under 1000. Not much use seems to have been made of these tables, both of which were published by the Commissioners of Longitude. Small tables for the conversion of sexagesimals into centesimals and vice versa are given in a few collections, such as Hülsse’s edition of Vega. H. Schubert’s *Fünfstellige Tafeln und Gegentafeln* (Leipzig, 1897) contains a sexagesimal table giving xy/60 for x = l to 59 and y = I to 150.

*Trigonometrical Tables (Natural).—*Peter Apian published in 1533 a table of sines with the radius divided decimally. The first complete canon giving all the six ratios of the sides of a right- angled triangle is due to Rheticus (1551), who also introduced the semiquadrantal arrangement. Rheticus’s canon was calcu­lated for every ten minutes to 7 places, and Vieta extended it to every minute (1579). In 1554 Reinhold published a table of tangents to every minute. The first complete canon published in England was by Thomas Blundeville (1594), although a table of sines had appeared four years earlier. Regiomontanus called his table of tangents (or rather cotangents) *tabula foecunda* on account of its great use; and till the introduction of the word “tangent” by Thomas Finck *(Geometriae rotundi libri XIV.,* Basel, 1583) a table of tangents was called a *tabula foecunda* or *canon foecundus.* Besides “ tangent," Finck also introduced the word "secant,” the table of secants having previously been called *tabula benefica* by Maurolycus (1558) and *tabula foecundissima* by Vieta.

By far the greatest computer of pure trigonometrical tables is George Joachim Rheticus, whose work has never been superseded. His celebrated ten-decimal canon, the *Opus palatinum,* was pub­lished by Valentine Otho at Neustadt in 1596, and in 1613 his fifteen-decimal table of sines by Pitiscus at Frankfort under the title *Thesaurus mathematicus.* The *Opus palatinum* contains a complete ten-decimal trigonometrical canon for every ten seconds of the quadrant, semiquadrantally arranged, with differences for all the tabular results throughout. Sines, cosines, and secants are given on the left-hand pages in columns headed respectively “Per- pendiculum,” “ Basis,’’ “ Hypotenusa,” and on the right-hand pages appear tangents, cosecants, and cotangents in columns headed respectively "Perpendiculum,” “ Hypotenusa," “ Basis.” At his death Rheticus left the canon nearly complete, and the trigonometry was finished and the whole edited by Valentine Otho; it was named in honour of the elector palatine Frederick IV., who bore the expense of publication. The *Thesaurus* of 1613 gives natural sines for every ten seconds throughout the quadrant, to 15 places, semiquadrantally arranged, with first, second, and third differences. Natural sines are also given for every second from 0° to 1° and from 89° to 90°, to 15 places, with first and second differences. The rescue of the manuscript of this work by Pitiscus forms a striking episode in the history- of mathematical tables. The alterations and emendations in the earlier part of the cor­rected edition of the *Opus palatinum* were made by Pitiscus, who had his suspicions that Rheticus had himself calculated a ten- second table of sines to 15 decimal places; but it could not be found. Eventually the lost canon was discovered amongst the papers of Rheticus which had passed from Otho to James Christ­mann on the death of the former. Amongst these Pitiscus found (1) the ten-second table of sines to 15 places, with first, second, and third differences (printed in the *Thesaurus)·,* (2) sines for every second of the first and last degrees of the quadrant, also to 15 places, with first and second differences; (3) the commence­ment of a canon of tangents and secants, to the same number of decimal places, for every ten seconds, with first and second differ­ences; (4) a complete minute canon of sines, tangents, and secants, also to 15 decimal places. This list, taken in connexion with the *Opus palatinum,* gives an idea of the enormous labours under­taken by Rheticus; his tables not only remain to this day the ultimate authorities but formed the data from which Vlacq calcu­lated his logarithmic canon. Pitiscus says that for twelve years Rheticus constantly had computers at work.

A history of trigonometrical tables by Charles Hutton was pre­fixed to all the early editions of his *Tables of Logarithms,* and forms Tract xix. of his *Mathematical Tracts,* vol. 1. p. 278, 1812. A good deal of bibliographical information about the *Opus palatinum* and earlier trigonometrical tables is given in A. De Morgan’s article “ Tables ” in the *English Cyclopaedia.* The invention of log­arithms the year after the publication of Rheticus’s volume by Pitiscus changed all the methods of calculation; and it is worthy of note that John Napier’s original table of 1614 was a logarithmic canon of sines and not a table of the logarithms of numbers. The logarithmic canon at once superseded the natural canon; and since Pitiscus’s time no really extensive table of pure trigono­metrical functions has appeared. In recent years the employment of calculating machines has reγived the use of tables of natural trigonometrical functions, it being found convenient for some purposes to employ such a machine in connexion with a natural canon instead of using a logarithmic canon. A. Junge’s *Tafel der wirklichen Länge der Sinus und Cosinus* (Leipzig, 1864) was pub­lished with this object. It gives natural sines and cosines for every ten seconds of the quadrant to 6 places. F. Μ. Clouth, *Tables pour le calcul des coordonnées goniométriques* (Mainz, n.d.), gives natural sines and cosines (to 6 places) and their first nine multiples (to 4 places) for every centesimal minute of the quadrant. Tables of natural functions occur in many collections, the natural and logarithmic values being sometimes given on opposite pages, sometimes side by side on the same page.

The following works contain tables of trigonometrical functions other than sines, cosines, and tangents. J. Pasquich, *Tabulae logarithmico-trigonometricae* (Leipzig, 1817), contains a table of sin2x, cos2x, tan2x, cot2x from x=1° to 45° at intervals' of 1' to 5 places. J. Andrew, *Astronomical and Nautical Tables* (London, 1805), contains a table of “ squares of natural semichords, *i.e.* of sin2½x from x = 0° to 120° at intervals of 10" to 7 places. This table was greatly extended by Major-General Hannyngton in his *Haversines, Natural and Logarithmic, used in computing Lunar Distances for the Nautical Almanac* (London, 1876). The name “ haversine,” frequently used in works upon navigation, is an abbreviation of “half versed sine”; viz., the haversine of x is equal to ½(1-cos x), that is, to sin2½x. The table gives logarithmic haversines for every 15" from 0° to 180°, and natural haversines for every 10" from 0° to 180°, to 7 places, except near the beginnng, where the logarithms are given to only 5 or 6 places. It occupies