327 folio pages, and was suggested by Andrew’s work, a copy of which by chance fell into Hannyngton’s hands. Hannyng- ton recomputed the whole of it by a partly\* mechanical method, a combination of two arithmometers being employed. A table of haversines is useful for the solution of spherical triangles when two sides and the included angle are given, and in other problems in spherical trigonometry. Andrew’s original table seems to have attracted very little notice. Hannyngton’s was printed, on the recommendation of the superintendent of the *Nautical Almanac* office, at the public cost. Before the calculation of Hannyngton’s table R. Farley’s *Natural Versed Sines* (London, 1856) was used in the *Nautical Almanac* office in computing lunar distances. This fine table contains natural versed sines from 0° to 125° at intervals of 10" to 7 places, with proportional parts, and log versed sines from 0° to 135° at intervals of 15" to 7 places. The argu­ments are also given in time. The manuscript was used in the office for twenty-five years before it was printed. Traverse tables, which occur in most collections of navigation tables, contain multiples of sines and cosines.

*Common or Briggian Logarithms of Numbers and Trigono­metrical Ratios.*—For an account of the invention and history of logarithms, see Logarithm. The following are the fundamental works which contain the results of the original calculations of logarithms of numbers and trigonometrical ratios:—Briggs, *Arith­metica logarithmica* (London, 1624), logarithms of numbers from I to 20,000 and from 90,000 to 100,000 to 14 places, with inter­script differences; Vlacq, *Arithmetica logarithmica* (Gouda, 1628, also an English edition, London, 1631, the tables being the same), ten-figure logarithms of numbers from 1 to 100,000, with differences, also log sines, tangents, and secants for every minute of the quad­rant to 10 places, with interscript differences; Vlacq, *Trigono­metries artificialis* (Gouda, 1633), log sines and tangents to every ten seconds of the quadrant to 10 places, with differences, and ten-figure logarithms of numbers up to 20,000, with differences; Briggs, *Trigonometria Britannica* (London, 1633), natural sines to 15 places, tangents and secants to 10 places, log sines to 14 places, and tangents to 10 places, at intervals of a hundredth of a degree from 0° to 45°, with interscript differences for all the functions. In 1794 Vega reprinted at Leipzig Vlacq’s two works in a single folio volume, *Thesaurus logarithmorum completus.* The arrange­ment of the table of logarithms of numbers is more compendious than in Vlacq, being similar to that of an ordinary seven-figure table, but it is not so convenient, as mistakes in taking out the differences are more liable to occur. The trigonometrical canon gives log sines, cosines, tangents, and cotangents, from 0° to 2° at intervals of one second, to 10 places, without differences, and for the rest of the quadrant at intervals of ten seconds. The trigonometrical canon is not wholly reprinted from the *Trigono- metrιa artificialis,* as the logarithms for every second of the first two degrees, which do not occur in Vlacq, were calculated for the work by Lieutenant Dorfmund. Vega devoted great attention to the detection of errors in Vlacq’s logarithms of numbers, and has given several important errata lists. F. Lefort *(Annales de l'Obser- vatoire de Paris,* vol. iv.) has given a full errata list in Vlacq’s and Vega’s logarithms of numbers, obtained by comparison with the great French manuscript *Tables du cadastre* (see Logarithm; comp. also *Monthly Notices R.A.S.,* 32, pp. 255, 288; 33, p. 330; 34, p. 447). Vega seems not to have bestowed on the trigono­metrical canon anything like the care that he devoted to the log­arithms of numbers, as Gauss@@1 estimates the total number of last-figure errors at from 31,983 to 47,746, most of them only amounting to a unit, but some to as much as 3 or 4.

A copy of Vlacq’s *Arithmetica logarithmica* (1628 or 1631), with the errors in numbers, logarithms, and differences corrected, is still the best table for a calculator who has to perform work requiring ten-figure logarithms of numbers, but the book is not easy to pro­cure, and Vega’s *Thesaurus* has the advantage of having log sines, &c., in the same volume. The latter work also has been made more accessible by a photographic reproduction by the Italian govern­ment *(Riproduzione fotozincografica dell' Islituto Geografico Mili- tare,* Florence, 1896). In 1897 Max Edler von Leber published tables for facilitating interpolations in Vega’s *Thesaurus (Tabularum ad faciliorem et breviorem in Georgii Vegae “ Thesauri logarith­morum ” magnis canonibus interpolationis computationem utilium Trias,* Vienna, 1897). The object of these tables is to take account of second differences. Prefixed to the tables is a long list of errors in the *Thesaurus,* occupying twelve pages. From an examination of the tabular results in the trigonometrical canon corresponding to 1060 angles von Leber estimates that out of the 90,720 tabular results 40,396 are in error by = 1,2793 by =2, and 191 by =3. Thus his estimated value of the total number of last-figure errors is 43,326, which is in accordance with Gauss’s estimate. A table of ten-figure logarithms of numbers up to 100,009, the result of a new calcula­tion, was published in the *Report of the U.S. Coast and Geodetic Survey for* 1895-6 (appendix 12, pp. 395-722) by W. W. Duffield,

superintendent of the survey. The table was compared with Vega's *Thesaurus* before publication.

S. Pincto’s *Tables de logarithmes vulgaires à dix décimales, con­struites d’après un nouveau mode* (St Petersburg, 1871), though a tract of only 80 pages, may be usefully employed when Vlacq and Vega arc unprocurable. Pineto’s work consists of three tables: the first, or auxiliary table, contains a scries of factors by which the numbers whose logarithms are required arc to be multiplied to. bring them within the range of table 2 ; it also gives the log­arithms of the reciprocals of these factors to 12 places. Table 1 merely gives logarithms to 1000 to 10 places. Tabic 2 gives logarithms from 1,000,000 to 1,011,000, with proportional parts to hundredths. The mode of using these tables is as follows. If the logarithm cannot be taken out directly from table 2, a factor *M* is found from the auxiliary table by which the number must be multiplied to bring it within the range of table 2. Then the logarithm can be taken out, and, to neutralize the effect of the multiplication, so far as the result is concerned, log 1/*M* must be added; this quantity is therefore given in an adjoining column to *M* in the auxiliary table. A similar procedure gives the number answering to any logarithm, another factor (approximately the reciprocal of *M*) being given, so that in both cases multiplication is used. The laborious part of the work is the multiplication by *M',* but this is somewhat compensated for by the ease with which, by means of the proportional parts, the logarithm is taken out. The factors are 300 in number, and are chosen so as to minimize the labour, only 25 of the 300 consisting of three figures all dif­ferent and not involving 0 or 1. The principle of multiplying by a factor which is subsequently cancelled by subtracting its log­arithm is used also in a tract, containing only ten pages, published by A. Namur and P. Mansion at Brussels in 1877 under the title *Tables de logarithmes à* 12 *décimales jusqu’à* 434 *milliards.* Here a table is given of logarithms of numbers near to 434,294, and other numbers are brought within the range of the table by multi­plication by one or two factors. The logarithms of the numbers near to 434,294 are selected for tabulation because their differ­ences commence with the figures 100 . . . and the presence of the zeros in the difference renders the interpolation easy.

The tables of S. Gundelfingcr and A. Nell *(Tafeln zur Berechnung neunstelliger Logarithmen,* Darmstadt, 1891) afford an easy means of obtaining nine-figure logarithms, though of course they are far less convenient than a nine-figure table itself. The method in effect consists in the use of Gaussian logarithms, viz., if N = n+*p*, log N=log *n*+log (1+p∕n) =log n+B where B is log *(1+p/n)* to argument A = log p-log *n.* The tables give log *n* from n=1000 to *n = 10,000,* and values of B for argument A.@@2

Until 1891, when the eight-decimal tables, referred to further on, were published by the French government, the computer who could not obtain sufficiently accurate results from seven-figure logarithms was obliged to have recourse to ten-figure tables, for, with only one exception, there existed no tables giving eight or nine figures. This exception is John Newton’s *Trigonometria Britannica* (London, 1658), which gives logarithms of numbers to 100,000 to 8 places, and also log sines and tangents for every centesimal minute *(i.e.* the nine-thousandth part of a right angle), and also log sines and tangents for the first three degrees of the quadrant to 5 places, the interval being the one- thousandth part of a degree. This table is also remarkable for giving the logarithms of the differences instead of the actual differ­ences. The arrangement of the page now universal in seven-figure tables—with the. fifth figures running horizontally along the top line of the page—is due to John Newton.

As a rule seven-figure logarithms of numbers are not published separately, most tables of logarithms containing both the logarithms of numbers and a trigonometrical canon. Babbage's and Sang’s logarithms are exceptional and give logarithms of numbers only. C. Babbage, *Table of the Logarithms of the Natural Numbers from I to* 108,000 (London, stereotyped in 1827; there are many tirages of later dates), is the best for ordinary use. Great pains were taken to get the maximum of clearness. The change of figure in the middle of the block of numbers is marked by a change of type in the fourth figure, which (with the sole exception of the asterisk) is probably the best method that has been used. Copies of the book were printed on paper of different colours—yellow·, brown, green, &c.—as it was considered that black on a white ground was a fatiguing combination for the eye. The tables were also issued with title-pages and introductions in other languages. In 1871 E. Sang published *A New Table of Seven-place Logarithms of all Numbers from* 20,000 to 200,000 (London). In an ordinary table extending from 10,000 to 100,000 the differences near the beginning are so numerous that the proportional parts arc either very crowded or some of them omitted; by making the table extend from 20,000 to 200,000 instead of from 10,000 to 100,000 the differences are halved in magnitude, while there are only one- fourth as many in a page. There is also greater accuracy. A

@@@1 See his “ Einige Bemerkungen zu Vega's *Thesaurus logarith­morum,”* in *Astronomische Nachrichten* for 1851 (reprinted in his *Werke,* vol. iii. pp. 257-64); also *Monthly Notices R.A.S.,* 33, p. 440.

@@@2 A seven-figure table of the same kind is contained in S. Gundel- finger’s *Sechsstellige Gaussische und siebenstellige gemeine Logarithmen* (Leipzig, 1902).