further peculiarity of this table is that multiples of the differences, instead of proportional parts, are given at the side of the page. Typographically the table is exceptional, as there are no rules, the numbers being separated from the logarithms by reversed commas —a doubtful advantage. This work was to a great extent the result of an original calculation; see *Trans. Roy. Soc. Edin.,* 1871, 26. Sang proposed to publish a nine-figure table from 1 to 1,000,000, but the requisite support was not obtained. Various papers of Sang’s relating to his logarithmic calculations will be found in the *Proc. Roy. Soc. Èdin.* subsequent to 1872. Reference should here be made to Abraham Sharp’s table of logarithms of numbers from I to 100 and of primes from 100 to 1100 to 61 places, also of numbers from 999,990 to 1,000,010 to 63 places. These first appeared in *Geometry Improv'd . . . by A. S. Philomath* (London, 1717). They have been republished in Sherwin’s, Callet’s, and the earlier editions of Hutton’s tables. H. Μ. Parkhurst, *Astronomical Tables* (New York, 1871), gives logarithms of numbers from 1 to 109 to 102 places.@@1

In many seven-figure tables of logarithms of numbers the values of 5 and *T* are given at the top of the page, with *V,* the variation of each, for the purpose of deducing log sines and tangents. S and *T* denote log (sin *x∕x)* and log (tan *x∕x)* respectively, the argument being the number of seconds denoted by certain numbers (some­times only the first, sometimes every tenth) in the number column on each page. Thus, in Callet’s tables, on the page on which the first number is 67200, S = log (sin 6720"/6720) and T = log (tan 6720"∕6720), while the V's are the variations of each for 10". To find, for example, log sin 1° 52'12"∙7, or log sin 6732\*∙7, we have S = 4·6854980 and log 6732∙7 =3,8281893, whence, by addition, we obtain 8·5136873; but *V* for 10" is - 2∙29, whence the variation for 12"∙7 is -3, and the log sine required is 8∙5136870. Tables of S and *T* are frequently called, after their inventor, “ Delambre’s tables.”

Some seven-figure tables extend to 100,000, and others to 108,000, the last 8000 logarithms, to 8 places, being given to ensure greater accuracy, as near the beginning of the numbers the differences are large and the interpolations more laborious and less exact than in the rest of the table. The eight-figure logarithms, however, at the end of a seven-figure table are liable to occasion error; for the computer who is accustomed to three leading figures, common to the block of figures, may fail to notice that in this part of the table there are four, and so a figure (the fourth) is sometimes omitted in taking out the logarithm. In the ordinary method of arranging a seven-figure table the change in the fourth figure, when it occurs in the course of the line, is a source of frequent error unless it is very clearly indicated. In the earlier tables the change was not marked at all, and the computer had to decide for himself, each time he took out a logarithm, whether the third figure had to be increased. In some tables the line is broken where the change occurs; but the dislocation of the figures and the corresponding irregularity in the lines are very awkward. Babbage printed the fourth figure in small type after *a* change; and Bremiker placed a bar over it. The best method seems to be that of prefixing an asterisk to the fourth figure of each logarithm after the change, as is done in Schrön’s and many other modern tables. This is beautifully clear and the asterisk at once catches the eye. Shortrede and Sang replace o after a change by a *nokta* (resembling a diamond in a pack of cards). This is very clear in the case of the o’s, but leaves unmarked the cases in which the fourth figure is 1 or 2. A method which' finds favour in some recent tables is to underline all the figures after the increase, or to place a line over them.

Babbage printed a subscript point under the last figure of each logarithm that had been increased. Schrön used a bar subscript, which, being more obtrusive, seems less satisfactory. In some tables the increase of the last figure is only marked when the figure is increased to a 5, and then a Roman five (v) is used in place of the Arabic figure.

Hereditary errors in logarithmic tables are considered in two papers “ On the Progress to Accuracy of Logarithmic Tables ” and “ On Logarithmic Tables,” in *Monthly Notices R.A.S.,* 33, pp. 330, 440. See also vol. 34, p. 447 ; and a paper by Gernerth, *Ztsch. f. d. österr.Gymm.,* Heft vi. p. 407.

Passing now to the logarithmic trigonometrical canon, the first great advance after the publication of the *Trigonometria artificialis* in 1633 was made in Michael Taylor’s *Tables of Logarithms* (London, 1792), which give log sines and tangents to every second of the quadrant to 7 places. This table contains about 450 pages with an average number of 7750 figures to the page, so that there are altogether nearly three millions and a half of figures. The change

in the leading figures, when it occurs in a column, is not marked at all; and the table must be used with very great caution. In fact it is advisable to go through the whole of it, and fill in with ink the first o after the change, as well as make some mark that will catch the eye at the head of every column containing a change. The table was calculated by interpolation from the *Trigonometria artificialis* to 10 places and then reduced to 7, so that the last figure should always be correct. Partly on account of the absence of a mark to denote the change of figure in the column and partly on account of the size of the table and a somewhat inconvenient arrangement, the work seems never to have come into general use. Computers have always preferred V. Bagay’s *Nouvelles Tables astronomiques et hydrographiques* (Paris, 1829), which also contains a complete logarithmic canon to every second. The change in the column is very clearly marked by a large black nucleus, surrounded by a circle, printed instead of o. Bagay’s work having become rare and costly was reprinted with the errors corrected. The reprint, how­ever, bears the original title-page and date 1829, and there appears to be no means of distinguishing it from the original work except by turning to one of the errata in the original edition and examining whether the correction has been made.

The only other canon to every second that has been published is contained in R. Shortrede’s *Logarithmic Tables* (Edinburgh). This work was originally issued in 1844 in one volume, but being dis­satisfied with it Shortrede issued a new edition in 1849 in two volumes. The first volume contains logarithms of numbers, anti­logarithms, &c., and the second the trigonometrical canon to every second. The volumes are sold separately, and may be regarded as independent works; they are not even described on their title­pages as vol. i. and vol. ii. Ths trigonometrical canon is very com­plete in every respect, the arguments being given in time as well as in arc, full proportional parts being added, &c. The change of figure in the column is denoted by a nokta, printed instead of O where the change occurs. The page is crowded and the print not very clear, so that Bagay is to be preferred for regular use.

Previous to 1891 the only important tables in which the quadrant is divided centesimally were J. P. Hobert and L. Ideler, *Nouvelles tables trigonométriques* (Berlin, 1799), and C. Borda and J. B. J. Delambre, *Tables trigonométriques décimales* (Paris, 1801). The former give, among other tables, natural and log sines, cosines, tangents, and cotangents, to 7 places, the arguments proceeding to 3° at intervals of 10" and thence to 50° at intervals of 1' (centesimal), and also natural sines and tangents for the first hundred ten-thousandths of a right angle to 10 places. The latter gives long sines, cosines, tangents, cotangents, secants, and cosecants from 0° to 3° at intervals of 10" (with full proportional parts for every second), and thence to 50° at intervals of 1' (centesimal) to 7 places. There is also a table of log sines, cosines, tangents, and cotangents from 0' to 10' at intervals of 10" and from 0° to 50° at intervals of 10' (centesimal) to 11 places. Hobert and Ideler give a natural as well as a logarithmic canon; but Borda and Delambre give only the latter. Borda and Delambre give seven-figure logarithms of numbers to 10,000, the line being broken when a change of figure takes place in it.

The tables of Borda and Delambre having become difficult to procure, and seven-figure tables being no longer sufficient for the accuracy required in astronomy and geodesy, the French govern­ment in 1891 issued an eight-figure table containing (besides log­arithms of numbers to 120,000) log sines and tangents for every ten seconds (centesimal) of the quadrant, the latter being extracted from the *Tables du cadastre* of Prony (see Logarithm). The title of this fine and handsomely printed work is *Service géographique de l'armée: Tables des logarithmes à huit décimales . . . publiées par ordre du ministre de la guerre* (Paris, Imprimerie Nationale, 1891). These tables are now in common use where eight figures are required.

In Brigg’s *Trigonometria Britannica* of 1633 the degree is divided centesimally, and but for the appearance in the same year of Vlacq’.s *Trigonometria artificialis,* in which the degree is divided sexagesimally, this reform might have been effected. It is clear that the most suitable time for making such a change was when the natural canon was replaced by the logarithmic canon, and Briggs took advantage of this opportunity. He left the degree unaltered, but divided it centesimally instead of sexagesimally, thus ensuring the advantages of decimal division (a saving of work in interpolations, multiplications, &c.) with the minimum of change. The French mathematicians at the end of the 18th century divided the right angle centesimally, completely changing the whole system, with no appreciable advantages over Briggs’s system. In act the centesimal degree is as arbitrary a unit as the nonagesima’ and it is only the non-centesimal subdivision of the degree that gives rise to incon­venience. Briggs’s example was followed by Roe, Oughtred, and other 17th-century writers; but the centes mal division of the degree seemed to have entirely passed out of ise, till it was revived by C. Bremiker in his *Logarithmisch-trigonometrische Tafeln mit fünf Decimalstellen* (Berlin, 1872, 10th ed. revised by A. Kallius, 1906). This little book of 158 pages gives a five-figure canon to every hundredth of a degree with proportional parts, besides logarithms of numbers, addition and subtraction logarithms, &c.

The eight-figure table of 1891 has now made the use of a cen­tesimal table compulsory, if this number of figures is required.

@@@1 Legendre *(Traité des fonctions elliptiques,* vol. ii., 1826) gives a table of natural sines to 15 places, and of log sines to 14 places, for every 15" of the quadrant, and also a table of logarithms of uneven numbers from 1163 to 1501, and of primes from 1501 to 10,000 to 19 places. The latter, which was extracted from the *Tables du cadastre,* is a continuation of a table in W. Gardiner's *Tables of Logarithms* (London, 1742; reprinted at Avignon, 1770), which gives logarithms of all numbers to 1000, and of uneven numbers from 1000 to 1143. Legendre's tables also appeared in his *Exercices de calcul integral,* vol. iii. (1816).