through several more. J. Galbraith and S. Haughton, *Manual of Mathematical Tables* (London, 1860), give five-figure logarithms to 10,000 and log sines and tangents for every minute, also a small table of Gaussian logarithms. J. Houël, *Tables de logarithmes à cinq décimales* (Paris, 1871 ; new edition 1907), is a very convenient collection of five-figure tables; besides logarithms of numbers and circular functions, there are Gaussian logarithms, least divisors of numbers to 10,841, antilogarithms, &c. The work (118 pp.) is printed on thin paper. A. Gernerth, *Fünfstellige gemeine Logarithmen (*Vienna, 1866), gives logarithms to 10,800 and a ten-second canon.

There are sixty lines on the page, so that the double page contains log sines, cosines, tangents, and cotangents extending over a minute. C. Bremiker, *Logarithmisch-trigonometrische Tafeln mit fünf Decimal- stellen* (10th edition by A. Kallius, Berlin, 1906), which has been already referred to, gives logarithms to 10,009 and a logarithmic canon to *every hundredth* of a degree (sexagesimal), in a handy volume; the lines are divided into groups of three, an arrangement about the convenience of which there is a difference of opinion. H. Gravelius, *Fünfstellige logarithmisch-trigonometrische Tafeln für die Decimalteilung des Quadranten* (Berlin, 1886), is a well-printed five-figure table giving logarithms to 10,009, a logarithmic canon to every centesimal minute (*i.e.* ten-thousandth part of a right angle), and an extensive table (40 pp.) for the conversion of centesimally expressed arcs into sexagesimally expressed arcs and vice versa. Among the other tables is a four-place table of squares from 0 to 10 at intervals of ∙001 with proportional parts. E. Becker, *Logarith­misch-trigonometrisches Handbuch auf fünf Decimalen* (2nd stereo. ed., Leipzig, 1897), gives logarithms to 10,009 and a logarithmic canon for every tenth of a minute to 6° and thence to 45° for every minute. There are also Gaussian logarithms. V. E. Gamborg, *Logaritmetabel* (Copenhagen, 1897), is a well-printed collection of tables, which contains a five-figure logarithmic canon to every minute, five-figure logarithms of numbers to 10,000, and five-figure antilogarithms, viz., five-figure numbers answering to four-figure mantissae from ∙0000 to ∙9999 at intervals of ∙0001. H. Schubert, *Fünfstellige Tafeln und Gegentafeln* (Leipzig, 1896), is peculiar in giving, besides logarithms of numbers and a logarithmic and natural canon, the three converse tables of numbers answering to logarithms, and angles answering to logarithmic and natural trigonometrical functions. The five-figure tables of F. G. Gauss (Berlin, 1870) have passed through very many editions, and mention should also be made of those of T. Wittstein (Hanover, 1859) and F. W. Rex (Stuttgart, 1884). S. W. Holman, *Computation Rules and Logarithms* (New York, 1896), contains a well-printed and convenient set of tables including five-figure logarithms of numbers to 10,000 and a five-figure logarithmic canon to every minute, the actual characteristics (with the negative sign above the number) being printed, as in the tables of Dupuis, 1868, referred to above. There is also a four-place trigonometrical canon and four-place anti­logarithms, reciprocals, square and cube roots, &c. G. W. Jones, *Logarithmic Tables* (4th ed., London, and Ithaca, N.Y., 1893), con­tains a five-place natural trigonometrical canon and a six-place logarithmic canon to every minute, six-place Gaussian and hyper­bolic logarithms, besides a variety of four-place tables, including squares, cubes, quarter-squares, reciprocals, &c. The factor table has been already noticed. It is to be observed that the fourth edition is quite a distinct work from the third, which contained much fewer tables. J. B. Dale, *Five-figure Tables of Mathematical Func­tions* (London, 1903), is a book of 92 pages containing a number of small five-figure tables of functions which are not elsewhere to be found in one volume. Among the functions tabulated are elliptic functions of the first and second kind, the gamma function, Legendre’s coefficients, Bessel’s functions, sine, cosine, and exponential integrals, &c. J. Houël's *Recueil de formules et de tables numériques* (Paris, 1868) contains 19 tables, occupying 62 pages, most of them giving results to 4 places; they relate to very varied subjects—anti­logarithms, Gaussian logarithms, logarithms of I+x/1 — *x* elliptic integrals, squares for use in the method of least squares, &c. C. Bremiker, *Tafel vierstelliger Logarithmen* (Berlin, 1874), gives four- figure logarithms, of numbers to 2009, log sines, cosines, tangents, and cotangents to 8° for every hundredth of a degree, and thence to 45° for every tenth of a degree, to 4 places. There are also Gaussian logarithms, squares from o·ooo to 13,500, antilogarithms, &c. The book contains 60 pages. It is not worth while to give a list of four- figure tables or other tables of small extent, which are very numerous, but mention may be made of J. Μ. Peirce, *Mathematical Tables chiefly to Four Figures* (Boston, U.S., 1879), 42 pp., containing also hyperbolic functions; W. Hall, *Four-figure Tables and Constants* (Cambridge, 1905), 60 pp., chiefly for nautical computation; A. du P. Denning, *Five-figure Mathematical Tables for School and Laboratory Purposes* (12 pp. *of* tables, large octavo); A. R. Hinks, *Cambridge Four-figure Mathematical Tables* (12 pp.). C. Μ. Willich, *Popular Tables* (London, 1853), is a useful book for an amateur; it gives Briggian and hyperbolic logarithms to 1200 to 7 places, squares, &c., to 343, &c.

*■Hyperbolic or Napierian or Natural Logarithms.—*The logarithms invented by Napier and explained by him in the *Descriptio* (1614) were not the same as those now called natural or hyperbolic (viz., to base e), and very frequently also Napierian, logarithms. Napierian logarithms, strictly so called, have entirely passed out of use and are of purely historic interest; it is therefore sufficient to refer to the article Logarithm, where a full account is given. Apart from the inventor’s own publications, the only strictly Napierian tables of importance are contained in Ursinus's *Trigonometria* (Cologne, 1624-1625) and Schulze’s *Sammlung* (Berlin, 1778), the former being the largest that has been constructed. Logarithms to the base e, where *e* denotes 2∙71828 . . ., were first published by J. Speidell, *New Logarithmes* (1619).

The most copious table of hyperbolic logarithms is Z. Dase, *Tafel der natürlichen Logarithmen* (Vienna, 1850), which extends from I to 1000 at intervals of unity and from 1000 to 10,500 at intervals of ·1 to 7 places, with differences and proportional parts, arranged as in an ordinary seven-figure table. By adding log 10 to the results the range is from 10,000 to 105,000 at intervals of unity. The table formed part of the *Annals of the Vienna Observatory* for 1851, but separate copies were printed. The most elaborate table of hyper­bolic logarithms is due to Wolfram, who calculated to 48 places the logarithms of all numbers up to 2200, and of all primes (also of a great many composite numbers) between this limit and 10,009. Wolfram’s results first appeared in Schulze’s *Sammlung* (1778). Six logarithms which Wolfram had been prevented from computing by a serious illness were supplied in the *Berliner Jahrbuch,* 1783, p. 191. The complete table was reproduced in Vega’s *Thesaurus* (1794), where several errors were corrected. Tables of hyperbolic logarithms are contained in the following collections:—Callet, all numbers to 100 and primes to 1097 to 48 places; Borda and Delambre (1801), all numbers to 1200 to 11 places; Salomon (1827), all numbers to 1000 and primes to 10,333 to 10 places; Vega, *Tabulae* (including Hülsse's edition, 1840), and Köhler (1848), all numbers to 1000 and primes to 10,000 to 8 places; Barlow (1814), all numbers to 10,000; Hutton, *Mathematical Tables,* and Willich (1853), all numbers to 1200 to 7 places; Dupuis (1868), all numbers to 1000 to 7 places. Hutton also gives hyperbolic logarithms from 1 to 10 at intervals of ∙01 to 7 places. *Rees's Cyclopaedia* (1819), art, “ Hyperbolic Logarithms,” contains a table of hyperbolic logarithms of all numbers to 10,000 to 8 places.

Logarithms to base *e* are generally termed *Napierian* by English writers, and *natural* by foreign writers. There seems no objection to the former name, though the logarithms actually invented by Napier depended on the base *e―1,* but it should be mentioned in text-books that so-called Napierian logarithms are not identical with those originally devised and calculated by Napier.

*Tables to convert Briggian into Hyperbolic Logarithms, and vice versa.—*Such tables merely consist of the first hundred (sometimes only the first ten) multiples of the modulus ∙43429 44819 . . . and its reciprocal 2∙30258 50929 . . . to 5, 6, 8, 10, or more places. They are generally to be found in collections of logarithmic tables, but rarely exceed a page in extent, and are very easy to construct. Schrön and Bruhns both give the first hundred multiples of the modulus and its reciprocal to 10 places, and Bremiker (in his edition of Vega and in his six-figure tables) and Dupuis to 7 places. C. F. Degen, *Tabularum Enneas* (Copenhagen, 1824), gives the first hundred multiples of the modulus to 30 places.

*Antilogarithms.—*In the ordinary tables of logarithms the natural numbers are integers, while the logarithms are incommensurable. In an antilogarithmic canon the logarithms are exact quantities, such as ∙00001, ∙00002, &c., and the corresponding numbers are incommensurable. The largest and earliest work of this kind is J. Dodson’s *Antilogarithmic Canon* (London, 1742), which gives numbers to 11 places corresponding to logarithms from 0 to 1 at intervals of ∙00001, arranged like a seven-figure logarithmic table, with interscript differences and proportional parts at the bottom of the page. This work was the only large antilogarithmic canon for more than a century, till in 1844 Shortrede published the first edition of his tables; in 1849 he published the second edition, and in the same year Filipowski's tables appeared. Both these works contain seven-figure antilogarithms: Shortrede gives numbers to logarithms from 0 to 1 at intervals of ∙00001, with differences and multiples at the top of the page, and H. E. Filipowski, *A Table of Antilogarithms* (London, 1849), contains a table of the same extent, the proportional parts being given to hundredths.

Small tables of antilogarithms to 20 places occur in several collections of tables, as Gardiner (1742), Callet, and Hutton. Four- and five-place tables are not uncommon in recent works, as *e.g.* in Houël (1871), Gamborg (1897), Schubert (1896), Holman (1896).

*Addition and Subtraction, or Gaussian Logarithms.—*The object of such tables is to give log (α=δ) by only one entry when log *a* and log *b* are given. Let

*A* =log x, B=log (1+x-1), C=log (1+x).

Leaving out the specimen table in Z. Leonelli’s *Théorie des loga­rithmes additionnels et déductifs* (Bordeaux, 1803), in which the first suggestion was made,@@1 the principal tables are the following: Gauss, in Zach’s *Monatliche Correspondent* (1812), gives *B* and *C* for argument *A* from 0 to 2 at intervals of ∙001, thence to 3∙40

@@@1 Leonelli’s original work of 1803, which is extremely scarce, was reprinted by J. Houël at Paris in 1875.