at intervals of ∙01, and to 5 at intervals of ·1, all to 5 places. This table is reprinted in Gauss’s *Werke,* vol. iii. p. 244. E. A. Matthiessen, *Tafel zur bequemem Berechnung* (Altona, 1818), gives *B* and *C* to 7 places for argument *A* from 0 to 2 at intervals of -0001, thence to 3 at intervals of ∙001, to 4 at intervals of ∙01, and to 5 at intervals of ·1; the table is not conveniently arranged. Peter Gray, *Tables and Formulae* (London, 1849, and “ Addendum,” 1870), gives *C* for argument *A* from —3 to —1 at intervals of ∙001 and from —1 to 2 at intervals of ∙0001, to 6 places, with propor­tional parts to hundredths, and log (1 — *x)* for argument *A* from —3 to — I at intervals of ∙001 and from 1 to ∙18999 at intervals of ∙0001, to 6 places, with proportional parts. J. Zech, *Tafeln der Additions­und Subtractions-Logarithmen* (Leipzig, 1849), gives *B* for argument *A* from 0 to 2 at intervals of ∙0001, thence to 4 at intervals of ·001 and to 6 at intervals of ∙01 ; also *C* for argument *A* from 0 to ∙0003 at intervals of ∙0000001, thence to ∙05 at intervals of ·000001 and to ∙303 at intervals of ∙00001, all to 7 places, with proportional parts. These tables are reprinted from Hülsse's edition of Vega (1849); the 1840 edition of Hülsse’s Vega con­tained a reprint of Gauss's original table. T. Wittstein, *Loga­rithmes de Gauss à sept décimales* (Hanover, 1866), gives *B* for argument *A* from 3 to 4 at intervals of ∙1, from 4 to 6 at intervals of ∙01, from 6 to 8 at intervals of ∙001, from 8 to 10 at intervals of ∙0001, also from 0 to 4 at the same intervals. In this hand­some work the arrangement is similar to that in a seven-figure logarithmic table. Gauss’s original five-place table was reprinted in Pasquich, *Tabulae* (Leipzig, 1817); Köhler, *Jerome de la Lande's Tafeln* (Leipzig, 1832), and *Handbuch* (Leipzig, 1848); and Galbraith and Haughton, *Manual* (London, 1860). Houël, *Tables de loga­rithmes* (1871), also gives a small five-place table of Gaussian logarithms, the addition and subtraction logarithms being separated as in Zech. Modified Gaussian logarithms arc given by J. H. T. Müller, *Vierstellige Logarithmen* (Gotha, 1844), viz., a four-place table of *B* and —log (1 —x-1) from *A* =0 to ∙03 at intervals of ∙0001, thence to ∙23 at intervals of ∙001, to 2 at intervals of ∙01, and to 4 at intervals of ·1; and by Shortrede, *Logarithmic Tables* (vol. i., 1849), viz., a five-place table of *B\_* and log (1+x) from *A=5* to 3 at intervals of ∙l, from *A* =3 to 2·7 at intervals of ∙01, to 1∙3 at intervals of ∙001, to 3 at intervals of ∙01, and to 5 at intervals of ∙1. Filipowski's *Antilogarithms* (1849) contains Gaussian log­arithms arranged in a new way. The principal table gives log (x+1) as tabular result for log *x* as argument from 8 to 14 at intervals of ∙001 to 5 places. Weidenbach, *Tafel um den Logarithmen . .* . (Copenhagen, 1829), gives log for argument *A* from ·382 to 2·002 at intervals of ∙001, to 3∙6 at intervals of ∙01, and to 5∙5 at intervals of ∙1 to 5 places. J. Houël’s *Recueil de formules et de tables numériques* (2nd ed., Paris, 1868) contains tables of logιo(x+ι), iog1057r^> and 1°gι<⅛⅛ from loS x~ -5 tθ -3 at in­tervals of ∙ι, from log x= — 3 to — 1 at intervals of ∙oι, from log x = —I to o at intervals of ∙001. F. W. Rex (*Fünfstellige Logarithmen-Tafeln,* Stuttgart, 1884) gives also a five-figure table I + *x , .*

of log and E. Hammer in his *Sechsstellige Tafel der Werthe*

*für jeden Wert des Arguments log x* (Leipzig, 1902) gives a six- figure table of this function from log x = 7 to 1∙99000, and thence to 1∙999700 to 5 places. S. Gundelfinger’s *Sechsstellige Gaussische und siebenstellige gemeine Logarithmen* (Leipzig, 1902) contains a table of log10 (1+x) to 6 places from log *x=* —2 to 2 at intervals of ∙001. G. W. Jones’s *Logarithmic Tables* (4th ed., London, and Ithaca, N.Y., 1893) contain 17 pages of Gaussian six-figure tables; the principal of which give log (1+x) to argument log *x* from log x = —2 ∙80 to 0 at intervals of ∙001, and thence to ∙1999 at intervals of ∙0001, and log (1—x-1) to argument log *x* from log x = ∙4 to ∙5 at intervals of ∙0001, and thence to 2∙8 at intervals of ∙001. Gaussian logarithms to 5 or 4 places occur in many collec­tions of five-figure or four-figure tables.

*Quadratic Logarithms.—*In a pamphlet *Saggio di tavole dei loga- ritmi quadratici* (Udine, 1885) Conte A. di Prampero has described a method of obtaining fractional powers (positive or negative) of any number by means of tables contained in the work. If abx = N, then χ~~J∙⅞⅛ς-⅛-~~~~a~~~~,~~

and if the logarithms are taken to be Briggian and a = ιoτ⅛ι and *b = 2,* then x = log10 log10N/log 2+ 10.

This quantity the author defines as the quadratic logarithm of *N* and denotes by *LqN.* It follows from this definition that LqNr = LqN+log10r∕log102. Thus the quadratic logarithms of *N* and N3 where s is any power (positive or negative) of 2 have the same mantissa.

A subsidiary table contains the values of the constant log10r∕log102 for 204 fractional values of *r.* The main table contains the values of 1000 mantissae corresponding to arguments *N, N½, N¼, . . .* (which all have the same mantissae). Among the argu­ments are the quantities 10∙0, 10∙1, 10∙2, . . . 99∙9 (the interval being ∙1) and 10∙00, 10∙01, . . . 10∙99 (the interval being ∙01). As an example, to obtain the value of 12⅔ we take from the first table the constant —0·584962, which belongs to ⅔, and entering the main table with 12 we take out the quadratic logarithm 10∙109937 which, by applying the constant, gives 9∙524975 the quadratic logarithm of the quantity required.

An appendix (*Tavola degli esponenti)* gives the Briggian loga­rithms of the first 57 numbers to the first 50 numbers as base, viz. logxN for N = 2, 3 57 and x = 2, 3,. . ., 50. The results

are generally given to 6 places.

*Logistic and Proportional Logarithms.—*In most collections of tables of logarithms a five-place table of logistic logarithms for every second to 1° is given. Logistic tables give log 3600 —log x at inter­vals of a second, x being expressed in degrees, minutes, and seconds. In Schulze (1778) and Vega (1797) the table extends to x=3600" and in Callet and Hutton to x = 5280". Proportional logarithms for every second to 3° (*i.e.* log 10,800 — log x) form part of nearly all collections of tables relating to navigation, generally to 4 places, sometimes to 5. Bagay, *Tables* (1829), gives a five-place table, but such arc not often to be found in collections of mathematical tables. The same remark applies to tables of proportional log­arithms for every minute to 24h, which give to 4 or 5 places the values of log 1440 —log x. The object of a proportional or logistic table, or a table of log a —log x, is to facilitate the calculation of proportions in which the third term is *a.*

*Interpolation Tables.—*All tables of proportional parts may be regarded as interpolation tables. C. Bremiker, *Tafel der Pro­portionalteile* (Berlin, 1843), gives proportional parts to hundredths of all numbers from 70 to 699. Schrön, *Logarithms,* contains an interpolation table giving the first hundred multiples of all numbers from 40 to 410. Sexagesimal tables, already described, are inter­polation tables where the denominator is 60 or 600. Tables of the values of binomial theorem coefficients, which arc required when second and higher orders of differences are used, are described below. W. S. B. Woolhouse, *On Interpolation, Summation, and the Adjustment of Numerical Tables* (London, 1865), contains nine pages of interpolation tables. The book consists of papers ex­tracted from vols. 11 and 12 of the *Assurance Magazine.*

*Dual Logarithms.—*This term was used by Oliver Byrne in his *Dual Arithmetic, Young Dual Arithmetician, Tables of Dual Loga­rithms,* &c. (London, 1863-67). A dual number of the ascending branch is a continued product of powers of 1∙1, 1∙01, 1∙001, &c., taken in order, the powers only being expressed; thus ∣ 6,9,7,8 denotes (1∙1)6(1∙01)9(1∙001)7(1∙0001)8, the numbers following the J being called dual digits. A dual number which has all but the last digit zeros is called a dual logarithm; the author uses dual logarithms in which there are seven ciphers between the ∣ and the logarithm. Thus since 1·00601502 is equal to ∣ 0,0,0,0,0,0,0,599702 the whole number 599702 is the dual logarithm of the natural number 1∙00601502.

A dual number of the descending branch is a continued product of powers of ∙9,∙99, &c. : for instance, (·9)3(∙99)3 is denoted by ’3 '2 ↑. The *Tables,* which occupy 112 pages, give dual numbers and log­arithms, both of the ascending and descending branches, and the corresponding natural numbers. The author claimed that his tables were superior to those of common logarithms.

*Constants.—*In nearly all tables of logarithms there is a page devoted to certain frequently used constants and their logarithms, such as ιr, ιr-1, ιrl, √7r. A specially good collection is printed in W. Templeton’s *Millwright's and Engineer’s Pocket Companion* (cor­rected by S. Maynard, London, 1871), which gives 58 constants involving ιr and their logarithms, generally to 30 places, and 13 others that may be properly called mathematical. A good list of constants involving π is given in Salomon (1827). A paper by G. Paucker in *Grunert's Archiv* ( vol. i. p. 9) has a number of constants involving *π* given to a great many places, and Gauss’s memoir on the lemniscate function *{Werke,* vol. iii.) has <r-\*, *e~l\*, e~i\*,* &c·, calculated to about 50 places. The quantity *π* has been worked out to 707 places (Shanks, *Proc. Roy. Soc.,* 21, p. 319).

J. C. Adams has calculated Euler's constant to 263 places *{Proc. Roy. Soc.,* 27, p. 88) and the modulus ∙43429 . . . to 272 places *{Id.* 42, p. 22). The latter value is quoted in *extenso* under Logarithm. J. Burgess on p. 23 of his paper of 1888, referred to under *Tables of ex,* has given a number of constants involving τ and p (the constant ∙476936 . . . occurring in the *Theory of Errors),* and their Briggian logarithms, to 23 places.

*Tables for the Solution of Cubic Equations.—*Lambert, *Supplementa* (1798), gives ±(x-x3) from x = ∙001 to 1∙155 as intervals of ∙001 to 7 places, and Barlow (1814) gives x3-x from x= 1 to 1∙1549 at intervals of ∙0001 to 8 places. Very extensive tables for the solution of cubic equations are contained in a memoir “ Beiträge zur Auflösung höherer Gleichungen ” by J. P. Kulik in the *Abh. der k. Böhm. Ges. der* *Wiss*. (Prague, 1860), 11, pp. 1-123. The principal tables (pp. 58-123) give to 7 (or 6) places the values of = (x—x3) from x=o to x = 3∙2800 at intervals of ∙001. There are also tables of the even and uneven determinants of cubic equations, &c. Other tables for the solution of equations are by A. S. Guldberg in the *Forhand.* of the *Videns-Selskab* of Christiania for 1871 and 1872 (equations of the 3rd and 5th order), by S. Gundelfinger, *Tafeln zur Berechnung der reellen Wurzeln sämtlicher trinomischen Gleichungen* (Leipzig, 1897), which depend on the use of Gaussian logarithms, and by R. Mehme, *Schlömilch's Zeitschrift,* 1898, 43, p. 80 (quadratic equations).