*Binominal Theorem Coefficients.—*Tables of the values of

x(x-1). x(x-ι)(x-2) x(x~l) ■ . ■ (x-5)

1.2 I .2.3 ' ' 1 .2 . . . 6

from x = ∙01 to x = ι at intervals of ∙oι to 7 places (which are useful in interpolation by second and higher orders of differences), occur in Schulze (1778)t Barlow (1814), Vega (1797 and succeeding editions), Hantschl (1827), and Köhler (1848). W. Rouse, *Doctrine of Chances* (London, n.d.), gives on a folding sheet (*a+b*)n for n≡l, 2,. . .20.

H. Gyldén (*Recueil des Tables,* Stockholm, 1880) gives binomial coefficients to n = 40 and their logarithms to 7 places. Lambert, *Supplementa* (1798), has the coefficients of the first 16 terms in (1+x)l and (1 —x)∙, their values being given accurately as decimals.

Vega (1797) has a page of tables giving ...—,... and

similar quantities to 10 places, with their logarithms to 7 places, and a page of this kind occurs in other collections. Köhler (1848) gives the values of 40 such quantities.

*Figurate Numbers.—*Denoting n(w+ι)... («·+£—ι)∕t! by [n],∙, Lambert, *Supplementa,* 1798, gives [»], from *n = l* to » = 30 and from i = l to t = i2; and G. W. Hill *(Amer. Jour. Math.,* 1884, 6, p. 130) gives logκ>[w]> for n = i, ', ⅞, J, ∣, and from *i* = 1 to t = 30.

*Trigonometrical Quadratic Surds.*—The surd values of the sines of every third degree of the quadrant are given in some tables of logarithms; *e.g.,* in Hutton’s (p. xxxix., ed. 1855), we find sin 35=⅛i√(5 + √5) + √⅛5∙+√a-√(i5+3√5)-^-√iii and the numerical values of the surds √ (5 + √5), √ (V)> &c„ are given to 10 places. These values were extended to 20 places by Peter Gray, *Mess. of Math.,* 1877, 6, p. 105.

*Circulating Decimals.—Cjoodwyn’s* tables have been described already. Several others have been published giving the numbers of digits in the periods of the reciprocals of primes : Burckhardt, *Tables des diviseurs du premier million* (Paris, 1814-1817), gave one for all primes up to 2543 and for 22 primes exceeding that limit. E. Desmarest, *Théorie des nombres* (Paris, 1852), included all primes up to 10,000. C. G. Reuschle, *Mathematische Abhandlung, enthaltend neue zahlentheoretische Tabellen* (1856), contains a similar table to 15,000. This W. Shanks extended to 60,000; the portion from I to 30,000 is printed in the *Proc. Roy. Soc.,* 22, p. 200, and the remainder is preserved in the archives of the society *(Id.,* 23, p. 260 and 24, p. 392). The number of digits in the decimal period of ι∕∕>, is the same as the exponent to which 10 belongs for modulus *p,* so that, whenever the period has *p — 1* digits, 10 is a primitive root of *p.* Tables of primes having a given number, *n,* of digits in their periods, *i.e.* tables of the resolutions of 10n-1 into factors and, as far as known, into prime factors, have been given by W. Looff (in *Grunert's Archiv,* 16, p. 54; reprinted in *Nouv. annales,* 14, p. 115) and by Shanks *(Proc. Roy. Soc.,* 22, p. 381). The former extends to re = 60 and the latter to *n* = 100, but there are gaps in both. Reuschle’s tract also contains resolutions of 10n —1.

There is a similar table by C. E. Bickmore in *Mess. of Math.,* 1896, 25, P∙ 43. A full account of all tables connecting *n* and *ρ* where 10n = 1, mod *p,* 10n being the least power for which this congruence holds good, is given by Allan Cunningham *(Id.,* 1904, 33, p. 145). The paper by the same author, “ Period-lengths of Circulates ” *(Id.* 1900, 29, p. 145) relates to circulators in the scale of radix *a.* See also tables of the resolutions of *an* — 1 into factors under *Tables relating to the Theory of Numbers* (below). Some further references on circulating decimals arc given in *Proc. Camb. Phil. Soc.,* 1878, 3, p. 185.

*Pythagorean Triangles.—*Right-angled triangles in which the sides and hypothenuse are all rational integers are frequently termed Pythagorean triangles, as, for example, the triangles 3, 4, 5, and 5, 12, 13. Schulze, *Sammlung* (1778), contains a table of such triangles subject to the condition tan ⅛ω7⅛(ω being one of the acute angles). About 100 triangles are given, but some occur twice. Large tables of right-angled rational triangles were given by C. A. Bretschneider, in *Grunert's Archiv,* 1841, 1, p. 96, and by Sang, *Trans. Roy. Soc. Edin.,* 1864, 33, p. 727. In these tables the triangles are arranged according to hypothenuses and extend to 1201, 1200, 49, and 1105, 1073, 264 respectively. W. A. Whitworth, in a paper read before the Lit. and Phil. Society of Liverpool in 1875, carried his list as far as 2465, 2337, 784. See also H. Rath, “ Die rationalen Dreiecke,” in *Grunert's Archiv,* 1874, 56, p. 188. Sang's paper also contains a table of triangles having an angle of 120° and their sides integers.

*Powers of* π.—G. Paucker, in *Grunert's Archiv,* p. 10, gives π-1 and *π½* to 140 places, and *τr~1,π-l,πi, ιrl* to about 50 places; J. Burgess *(Trans. Roy. Soc. Edin.,* 1898, 39, II., No. 9, p. 23) gives (⅜τr)-i, *2lπ~i,* and some other constants involving *π* as well as their Briggian logarithms to 23 places, and in Maynard's list of constants (see *Constants,* above) 1r2 is given to 31 places. The first twelve powers of *τr* and *π~, to* 22 or more places were given by Glaisher, in *Proc. Load. Math. Soc.,* 8, p. 140, and the first hundred multiples of *π* and ιr"1 to 12 places by J. P. Kulik, *Tafel der Quadrat- und Kubik-Zahlen* (Leipzig, 1848).

*The Series* ι^n+2^"+3^τ,+‰.—Let Sn, *sn, σn* denote respectively the sums of the series ι^l4-2-"+3^"-j- &c., I-" —2^n-∣-3~n-&c·, l^n+3-n-∣-5-"4- &c. Legendre *(Traité des fonctions elliptiques,* vol. 2, p. 432) computed 5„ to 16 places from *n = ι* to 35, and Glaisher *(Proc. Lond. Math. Soc.,* 4, p. 48) deduced snand *an* tor the same arguments and to the same number of places. The latter also gave Sn, sn, *an for n* = 2, 4, 6,.. .12 to 22 or more places *(Proc. Lond. Math. Soc.,* 8, p. 140), and the values of ∑n, where *∑n = 2~n+3~n +* 5“"+&c. (prime numbers only involved), for n = 2, 4, 6, ... 36 to 15 places *(Compte rendu de ΓAssoc. Française,* 1878, p. 172).

C. W. Merrifield *(Proc. Roy. Soc.,* 1881, 33, p. 4) gave the values of log, Sn and ∑n for n = l,2,3,...1 35 to 15 places, and Glaisher *(Quar. Jour. Math.,* 1891, 25, ρ. 347) gave the values of the same quantities for *n* =2,4,6,..., 80 to 24 places (last figure uncertain). Merrifield’s table was reprinted by J. P. Gram on p. 269 of the paper of 1884, referred to under *Sine-integral, δfc.,* who also added the values of Iog∣o5n for the same arguments to 15 places. An error in ∑s in Merrifield's table is pointed out in *Quar. Jour. Moth.,* 25, P∙ 373∙ This quantity is correctly given in Gram's reprint. T. J. Stielies has greatly extended Legendre’s table of *S„.* His table *(Acta math.,* 1887, 10, p. 299) gives 5n for all values of *n* up to » = 70 to 32 places. Except for six errors of a unit in the last figure he found Legendre’s table to be correct. Legendre's table was re­printed in De Morgan’s *Diff. and Int. Calc.* (1842), ρ. 554. Various small tables of other series, involving inverse powers of prime numbers, such as 3^^n-5~n+7-" + lln-13"+..., are given in vols. 25 and 26 of the *Quar. Jour. Math.*

*Tables of ex and e-x, or Hyperbolic Antilogarithms.—*The largest tables are the following: C. Gudermann, *Theorie der potenzial- oder cyklisch-hyperbolischen Functionen* (Berlin, 1833), which consists of papers reprinted from vols. 8 and 9 of *Crelle's Journal,* and gives ogιo sinh x, logιo cosh x, and logιo tanh x from x = 2 to 5 at intervals of ∙ooι to 9 places and from *x* = 5 to 12 at intervals of ∙oι to 10 places. Since sinh *x = i(ez-e^x)* and cosh *x≈i(ex+e^x),* the values of *ex* and *e~z* are deducible at once by addition and subtraction. F. W. Newman, in *Camb. Phil. Trans.,* 13, p. 145, gives values of *e~x* from x=o to 15∙349 at intervals of ∙ooι to 12 places, from *x* = 15∙350 to 17∙298 at intervals of ∙002, and from *x* = 17-300 to 27∙635 at intervals of ∙005, to 14 places. Glaisher, in *Camb. Phil. Trans.,* 13, p. 243, gives four tables of *ex, e^x,* logιoe\*, log∣o *e^x,* their ranges being from x = ∙ooι to ∙ι at intervals of ∙ooι, from ∙oι to 2 at intervals of ∙oι, from -I to 10 at intervals of ∙I, from 1 to 500 at intervals of unity. Vega, *Tabulae* (1797 and later ed.), has log∣o e\*to 7 places and e1to 7 figures from x = ∙oι to 10 at intervals of ∙01. Köhler’s *Handbuch* contains a small table of *ex.* In Schulze’s *Sammlung* (1778) e1is given for x = ι, 2, 3,... 24 to 28 or 29 figures and for x=25, 30, and 60 to *32* or 33 figures; this table is reprinted in Glaisher’s paper (loc. cιt.). In Salomon’s *Tafeln* (1827) the values of *en, e'n,* e-ω∙, e∙°0n,... e∙°00000n, where *n* has the values 1, 2, 9,

are given to 12 places. Bretschneider, in *Grunert's Archiv,* 3, p. 33, gavee1 and e^at and also sin χ and cos *x* for x = ι, 2,...10 to 20 places, and J. P. Gram (in his paper of 1884, referred to under *Sine­integral, &c.),* gives *ex* for x = ιo, ιι,...20 to 24 places, and from x=7 to x = 20 at intervals of 0∙2 to 10, 13, 14, or 15 places. J. Burgess *(Trans. Roy. Soc. Edin.,* 1888,39,11. No. 9) has given (p. 26)

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the values of *e~x* and *y∣-e~x* for x = ⅛ and for x = 1, 2,..., 10 to 30 *’ TΓ*

*2* places. In the same paper he also gives the values of ∙^~e~xi from

x = o tox = ι∙250 to 9 places, and from x = 1∙25 tox = ι∙50at intervals of ∙oι, and thence at various intervals to x = 6 to 15 places, and the

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values of log™ *j-e~x2* from x=ι to x = 3 at intervals of ∙001 to *’ ΊΓ*

16 places.

*Factorials.—*The values of log10 (n!), where *n!* denotes 1 .2.3... *n,* from *n* = 1 to 1200 to 18 places, are given by C. F. Degen, *Tabu­larum Enneas* (Copenhagen, 1824), and reprinted, to 6 places, at the end of De Morgan's article “ Probabilities ” in the *Encyclopaedia Metropolitana.* Shortrede, *Tables* (1849, vol. i.), gives log *(n!)* to *n* = 1000 to 5 places, and for the arguments ending in 0 to 8 places. Degen also gives the complements of the logarithms. The first 20 figures of the values of *nXn!* and the values of —log (n×n!) to 10 places are given by Glaisher as far as *n* = 71 in the *Phil. Trans.* for 1870 (p. 370), and the values of 1∕n! to 28 significant figures as far as *n* =50 in *Camb. Phil. Trans.,* 13, p. 246.

*Bernoullian Numbers.—*The first fifteen Bernoullian numbers were given by Euler, *Inst. Calc. Diff.,* part ii. ch. v. Sixteen more were calculated by Rothe, and the first thirty-one were published by Μ. Ohm in *Crelle’s Journal,* 20, p. 11. J. C. Adams calculated the next thirty-one, and a table of the first sixty-two was published by him in the *Brit. Ass. Report* for 1877 and in *Crelle's Journal,* 85, p. 269. In the *Brit. Ass. Report* the numbers arc given not only as vulgar fractions, but also expressed in integers and circulating decimals. The first nine figures of the values of the first 250 Bcr- noullian numbers, and their Briggian logarithms to 10 places, have been published by Glaisher, *Camb. Phil. Trans.,* 12, p. 384.

*Tables of log tan* (¼π∙-∣-⅜φ).—C. Gudermann, *Theorie der potenzial- oder cyklisch-hyperbolischen Functionen* (Berlin, 1833), gives (in 100 pages) log tan (iπ-HΦ) for every centesimal minute of the quadrant to 7 places. Another table contains the values of this function.