also at intervals of a minute, from 88° to 100° (centesimal) to 11 places. A. Μ. Legendre, *Traité des fonctions elliptiques* (vol. ii. p. 256), gives the same function for every half degree (sexagesimal) of the quadrant to 12 places.

*The Gamma Function.—*Legendre's great table appeared in vol. ii. of his *Exercices de calcul intégral* (1816), p. 85, and in vol. ii. of his *Traité des fonctions elliptiques* (1826), p. 489. Log10 Γ(x) is given from *x =* I to 2 at intervals of ∙001 to 12 places, with differences to the third order. This table is reprinted in full in O. Schlömilch, *Analytische Studien* (1848), p. 183; an abridgment in which the arguments differ by ∙01 is given by De Morgan, *Diff. and Int. Calc.,* p. 587. The last figures of the values omitted are also supplied, so that the full table can be reproduced. A seven-place abridgment (without differences) is published in J. Bertrand, *Calcul intégral* (1870), p. 285, and a six-figure abridgment in B. Williamson, *Integral Calculus* (1884), p. 169. In vol. i. of his *Exercices* (1811), Legendre had previously published a seven-place table of log10 Γ(x), without differences.

*Tables connected with Elliptic Functions.—*Legendre published elaborate tables of the elliptic integrals in vol. ii. of his *Traité des fonctions elliptiques* (1826). Denoting the modular angle by *0,* the amplitude by *Φ,* the incomplete integral of the first and second kind by *F(φ)* and *F1(Φ),* and the complete integrals by *K* and *E,* the tables are:—(1) log10E and log10K from 0 = 0° to 90° at intervals of 0°∙1 to 12 or 14 places, with differences to the third order; (2) E1(φ) and *F(Φ),* the modular angle being 45°, from *φ=0°* to 90° at intervals of 0°∙5 to 12 places, with differences to the fifth order; (3) E1(45°) and *F* (45°) from 0=0° to 90° at intervals of 1°, with differences to the sixth order, also *E* and *K* for the same arguments, all to 12 places; (4) E1(φ) and *F(φ)* for every degree of both the amplitude and the argument to 9 or 10 places. The first three tables had been published previously in vol. iii. of the *Exercices de calcul intégral* (1816).

*Tables involving q.—*P. F. Verhulst, *Traité des fonctions elliptiques* (Brussels, 1841), contains a table of log10(logw)j^1 for argument *0* at intervals of 0°∙1 to 12 or 14 places. C. G. J. Jacobi, in *Crelle's Journal,* 26, p. 93, gives log10 *q* from *0* =0° to 90s at intervals of 0°∙ 1 to 5 places. E. D. F. Meissel’s *Sammlung mathematischer Tafeln,* i. (Iserlohn, i860), consists of a table of log10 *q* at intervals of 1' from 0=0° to 90° to 8 places. Glaisher, in *Month. Not. R.A.S.,* 1877, 37∙ P∙ 372, gives log10 *q* to 10 places and *q* to 9 places for every degree. In J. Bertrand’s *Calcul Intégral* (1870), a table of log10 *q* from 0 = 0° to 90° at intervals of *5'* to 5 places is accompanied by tables of log10 √(2K∕τ) and log10 log10 q-1 and by abridgments of Legendre’s tables of the elliptic integrals. O. Schlömilch, *Vorlesungen der höheren Analysis* (Brunswick, 1879), p. 448, gives a small table of log10 *q* for every degree to 5 places.

*Legendrian Coefficients (Zonal Harmonies).—*The values of *Pn(x)* for *n* ≈l, 2, 3,.. .7 from *x = 0* to 1 at intervals of ∙01 are given by Glaisher, in *Brit. Ass. Rep.,* 1879, pp. 54-57. The functions tabulated are P1(x)=x, *Pi(x)* = ⅛(3x2-ι), P3(x) = K5x3-3x), F4(x) = i(35xt- 30x2+3), *P"* (x) = i (63x6 - 70xs+15x), *Pi* (x) = √β (231 xβ - 3*15x,* +105x2 -5), F7(x) =1∖(429x7-693x6+3i5x,-35x).

The values of Pn(cos 0) for π = ι, *2,..* .7 for 0 = o , I , 2 ,.. .90 to 4 places are given by J. Perry in the *Proc. Phys. Soc.,* J892, il, p. 221, and in the *Phil. Mag.,* 1891, ser. 6, 32, p. 512. The functions *P^* occur in connexion with the theory of interpolation, the attraction of spheroids, and other physical theories.

*Bessel's Functions.—*F. W. Bessel’s original table appeared at the end of his memoir, “ Untersuchung des planetarischen Teils der Störungen, welche aus der Bewegung der Sonne entstehen ” (in *Abh. d. Berl. Akad.* 1824; reprinted in vol. i. of his *Abhandlungen,* p. 84). It gives Λ(x) and ∙Λ(x) from x = o to 3∙2 at intervals of ∙01. More extensive tables were calculated by P. A. Hansen in “ Ermit­telung der absoluten Störungen in Ellipsen von beliebiger Excen- tricität und Neigung "· (in *Schriften der Sternwarte Seeberg,* part i., Gotha, 1843). They include an extension of Bessel's original table to x = 20, besides smaller tables of Λ>(x) for certain values of *n* as far as n=28, all to 7 places. Hansen’s table was reproduced by O. Schlömilch, in *Zeitschr. für Math.,* 2, p. 158, and by E. Lommel, *Studien über die Bessel'schen Functionen* (Leipzig, 1868), p. 127. Hansen’s notation is slightly different from Bessel’s; the change amounts to halving each argument. Schlömilch gives the table in Hansen's form; Lommel expresses it in Bessel's.

Lord Rayleigh's *Theory of Sound* (1894), 1, p. 321, gives J0(x) and J1(x) from *x = o* to x=13∙4 at intervals of 0∙1 to 4 places, taken from Lommel. A large table of the same functions was given by E. D. F. Meissel in the *Abh. d. Berlin Akad.* for 1888 (published also separately). It contains the values of Jo(x) and *J1(x)* from x = o to x = 15∙50 at intervals of ∙01. A. Lodge has calculated the values of the function In(x) where

Zn(x) - *i Jn(ix) — 2nn∣* j I +2(2w-f-2) +2-4. (2il -∣-2) (2W +4) -∣- - · · |

His tables give Zn(x) for n=o, ι, 2,..., 11 from x=o to x = 6 at intervals of 0∙2 to 11 or 12 places *(Brit. Ass. Rep.,* 1889, p. 29), *. I·/.x)* and *Ia(x)* from x = 0 to x=5∙100 at intervals of ∙001 to 9 places *(Id.,* 1893, p. 229, and 1896, p. 99), and of J0(x√i) from x=o to x = 6 at intervals of 0∙2 *(Id.,* 1893, p. 228) to 9 places. In all the tables the last figure is uncertain. Subsidiary tables for the calculation of Bessel’s functions are given by L. N. G. Filon and A. Lodge in *Brit. Ass. Rep.,* 1907, p. 94. The work is being continued, the object being to obtain the values of Jn(x) for n =o, ½ 1, 1½,.. ., 6½. A table by E. Jahnke has been announced, which, besides tables of other mathematical functions, is to contain values of Bessel's functions of order ⅓ and roots of functions derived from Bessel’s functions.

*Sine, Cosine, Exponential, and Logarithm Integrals.*—The func­tions so named arc the integrals í 2!E2⅛χ, ∣ -0-s xdx, í *e^dx, fo* ~~l<¾'x~~' wh,ch are denotc<i by the functional signs Si x, Ci x, Ei x, li x respectively, so that Ei x = li *ex.* J. von Soldner, *Théorie et tables d’une nouvelle fonction transcendante* (Munich, 1809), gave the values of li x from x=o to 1 at intervals of ∙l to 7 places, and thence at various intervals to 1220 to 5 or more places. This table is reprinted in De Morgan’s *Diff. andIni. Calc.,* p. 662. Bretschncider, in *Grunert's Archiv, 3,* p. 33, calculated Ei (=fcx), Si x, Ci x for x= 1,2, ... 10 to 20 places, and subsequently (in Schlömilch’s *Zeitschrift,* 6) worked out the values of the same functions from x = o to 1 at intervals of ∙oι and from 1 to 7∙5 at intervals of ∙ι to 10 places. Two tracts by L. Stenberg, *Tabulae logarithmi integralis* (Malmö, part i. ι86ι and part ii. 1867), give the values of li io\* from *x =* —15 to 3∙5 at intervals of ∙oι to 18 places. Glaisher, in *Phil. Trans.,* 1870, p. 367, gives Ei (=tx), Si x, Ci x from x = oto 1 at intervals of ∙oι to 18 places, from x = 1 to 5 at intervals of ∙l and thence to 15 at intervals of unity, and for x = 20 to 11 places, besides seven-place tables of Si x and Ci x and tables of their maximum and minimum values. Sec also Bellavitis, “ Tavole numeriche logaritmo-integrale ’’ (a paper in *Memoirs of the Venetian Institute,* 1874). F. W. Bessel calculated the values of li 1000, li 10,000, li 100,000, li 200,000,... li 600,000, and li 1,000,000 . (see *Abhandlungen,* 2, p. 339). In Glaisher, *Factor Table for the Sixth Million* (1883), § iii., the values of li x are given from x=o to 9,000,000 at intervals of 50,000 to the nearest integer. J. P. Gram in the publications of the Copenhagen Academy, 1884, 2, No. 6 (pp. 268-272), has given to 20 places the values of Ei x from x = 10 to x = 20 at intervals of a unit (thus carry­ing Bretschneider’s table to this extent) an<l to 8, 9, or 10 places, the values of the same function from x = 5 to x = 20 at intervals of 0∙2 (thus extending Glaishcr’s table in the *Phil. Trans.).*

*Values of ^e~x2dx and* ex2 J"θ e-τ2dx.—These- functions are em­ployed in researches connected with refractions, theory of errors, conduction of heat, &c. *LctJ^r e~τ,dx* and *Ç e~xidx* be denoted by erf x and erfc x respectively, standing for “ error function ” and “error function complement?' so that erf x-∣-erfc x = i√τ *(Phil. Mag.,* Dec. 1871; it has since been found convenient to transpose as above the definitions there given of erf and erfc). The tables of the functions, and of the functions multiplied by eil, are as follows. C. Kramp, *Analyse des Réfractions* (Strasbourg, 1798), has erfc x from x=o to 3 at intervals of ∙oι to 8 or more places, also logw (erfc x) and logw (e\*%rfc x) for the same values to 7 places. F. W. Bessel, *Fundamenta astronomiae* (Königsberg, 1818), has logw (e\*2crfc x) from x = 0 to I at intervals of ∙oι to 7 places, likewise for argument logw x, the arguments increasing from o to 1 at intervals of ∙oι. A. Μ. Legendre, *Traité des fondions elliptiques* (1826), 2, p. 520, contains Γ(J, e—x2), that is, 2 erfc x from x = o to ∙5 at intervals of ∙oι to 10 places. J. F. Encke, *Berliner ast. Jahrbuch* for 1834, gives erf x from x=o to 2 at intervals of ∙01 to 7 places and erf (ρx) from x = o to 3∙4 at intervals of ∙oι and thence to 5 at intervals of ∙ι to 5ιplaccs, *p* being ∙4769360. Glaisher, in *Phil. Mag.,* December 1871, gives erfc x from x = 3 to 4∙5 at intervals of ∙01 to 11, 13, or 14 places. Encke’s tables and two of Kramp’s were reprinted in the *Encyclo­paedia Metropolitana,* art. “ Probabilities.” These tables have also been reprinted in many foreign works on probabilities, errors of observations, &c. Γn vol. 2 (1880) of his *Lenrbuch zur Bahnbestim­mung der Kometen und Planeten* T. R. v. Oppolzer gives (p. 587) a table of erf x from x = o to 4∙52 at intervals of ∙01 to 10 places, and

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(p. 603) a table of erf x from x=o to 2 at intervals of ∙oι to 5 places. Both tables were the result of original calculations. A very large table of logw *ext* erfc x was calculated by R. Radau and published in the *Annales de l'observatoire de Paris (Mémoires,* 1888, 18, B. 1-25). It contains the values of logw e\*2 erfc x from.x— —0∙120 to ι∙ooo at intervals of ∙ooι to 7 places, with differences. A. Markoff in a separate publication, *Table des valeurs de l’intégrale J" e~adl* (St Petersburg, 1888), gives erfc x from x = o to 3 at intervals of ∙ooι and from x = 3 to 4∙80 at intervals of ∙oι, with first, second, and third differences to 11 places. He also gives a table of erf x from x = o to x = 2∙499 at intervals of ∙ooι and thence to 3∙79 at intervals of ∙oι. J. Burgess, *Trans. Roy. Soc. Edin.,* 1888, 39, IL, No. 9, 2

published very extensive tables of erf *x,* which were entirely