that he discovered many things himself, and communicated the beginnings of many to his successors, some of which he attempted in a more abstract manner *(καΟολικύτηρον)* and some in a more intuitional or sensible manner (αiσOητικωτeρor) *(op. cit.* p. 65).

From these indications it is no doubt difficult to determine what Thales brought from Egypt and what was due to his own inven­tion. This difficulty has, however, been lessened since the trans­lation and publication of the papyrus Rhind by Eisenlohr;@@l and it is now generally admitted that, in the distinction made in the last passage quoted above from Proclus, reference is made to the two forms of his work―*alσθητικiώτepoν* pointing to what he derived from Egypt or arrived at in an Egyptian manner, while *κaθoλiκωτϵpov* indicates the discoveries which he made in accord­ance with the Greek spirit. To the former belong the theorems (1), (2), and (3), and to the latter especially the theorem (4), and also, probably, his solution of the two practical problems. We infer, then, [1] that Thales must have known the theorem that the sum of the three angles of a triangle are equal to two right angles. This inference is made from (4) taken along with (2). No doubt we are informed by Proclus, on the authority of Eudemus, that the theorem Euclid i. 32 was first proved in a general way by the Pythagoreans; but, on the other hand, we learn from Geminus that the ancient geometers observed the equality to two right angles in each kind of triangle—in the equilateral first, then in the isosceles, and lastly in the scalene (Apoll. *Conica,* ed. Halleius, p. 9), and it is plain that the geometers older than the Pythagoreans can be no other than Thales and his school. The theorem, then, seems to have been arrived at by induction, and may have been suggested by the contemplation of floors or walls covered with tiles of the form of equilateral triangles, or squares, or hexagons. [2] We see also in the theorem (4) the first trace of the important conception of geometrical loci, which we, there­fore, attribute to Thales. It is worth noticing that it was in this manner that this remarkable property of the circle, with which, in fact, abstract geometry was inaugurated, presented itself to the imagination of Dante:—

“ O se del mezzo cerchio far si puote

Triangol sì, ch’un retto non avesse.”—*Par. c.* xiii. 101.

[3] Thales discovered the theorem that the sides of equiangular triangles are proportional. The knowledge of this theorem is dis­tinctly attributed to Thales by Plutarch, and it was probably made use of also in his determination of the distance of a ship at sea.

Let us now consider the importance of the work of Thales.

I. In a scientific point of view: *(a)* we see, in the first place, that by his two theorems he founded the geometry of lines, which has ever since remained the principal part of geometry; *(b)* he may, in the second place, be fairly considered to have laid the founda­tion of algebra, for his first theorem establishes an equation in the true sense of the word, while the second institutes a proportion.@@2

II. In a philosophic point of view: we see that in these two theorems of Thales the first type of a natural law, *i.e.* the expression of a fixed dependence between different quantities, or, in another form, the disentanglement of constancy in the midst of variety—has decisively arisen.@@3 Ill. Lastly, in a practical point of view: Thales furnished the first example of an application of theoretical geometry to practice,@@4 and laid the foundation of an important branch of the same—the measurement of heights and distances. For the further progress of geometry see Pythagoras.

As to the astronomical knowledge of Thales we have the follow- ing notices:—(1) besides the prediction of the solar eclipse, Eudemus attributes to him the discovery that the circuit of the sun between the solstices is not always uniform;@@5 (2) he called the last day of the month the thirtieth (Diog. Laër. i. 24) ; (3) he divided the year into 365 days *(Id.* i. 27); (4) he determined the diameter of the sun to be the 720th part of the zodiac;@@6 (5) he appears to have pointed out the constellation of the Lesser Bear to his countrymen, and instructed them to steer by it [as nearer the pole] instead of the Great Bear (Callimachus ap. Diog. Laër. i 23; cf. Aratus, *Phaenomena,* v. 36 seq.). Other discoveries in astronomy are attributed to Thales, but on authorities which are not trustworthy. He did not know, for example, that “ the earth is spherical,” as is erroneously stated by Plutarch *(Placita,* iii. 10); on the contrary, he conceived it to be a flat disk, and in this sup­position he was followed by most of his successors in the Ionian schools, including Anaxagoras. The doctrine of the sphericity oí

the earth, for which the researches of Anaximander had prepared the way,@@7 was in fact one of the great discoveries of Pythagoras, was taught by Parmenides, who was connected with the Pytha­goreans, and remained for a long time the exclusive property of the Italian schools.@@8 (G. J. A.)

*Philosophy.—*Whilst in virtue of his political sagacity and intellectual eminence Thales held a place in the traditional list of the wise men, on the strength of the disinterested love of knowledge which appeared in his physical speculations he was accounted a “ philosopher ” (φιλόσοφοί). His “ philosophy ” is usually summed up in the dogma “ water is the principle, or the element, of things but, as the technical terms“ principle ” (άρχή) and “ element ” (σroιχeιoν) had not yet come into use, it may be conjectured that the phrase “ all things are water” *(πάντα υδωρ* cστf) more exactly represents his teaching. Writings which here his name were extant in antiquity; but as Aristotle, when he speaks of Thales’s doctrine, always depends upon tradition, there can be little doubt that they were forgeries.

From Aristotle we learn (1) that Thales found in water the origin of things; (2) that he conceived the earth to float upon a sea of the elemental fluid; (3) that he supposed all things to be full of gods; (4) that in virtue of the attraction exercised by the magnet he attributed to it a soul. Here our information ends. Aristotle’s suggestion that Thales was led to his fundamental dogma by observation of the part which moisture plays in the production and the maintenance of life, and Simplicius’s, that the impressibility and the binding power of water were perhaps also in his thoughts, are by admission purely conjectural. Simplicius’s further suggestion that Thales conceived the ele­ment to be modified by thinning and thickening is plainly inconsistent with the statement of Theophrastus that the hypo­thesis in question was peculiar to Anaximenes. The assertion preserved by Stobaeus that Thales recognized, together with the material element “ water,” “ mind,” which penetrates it and sets it in motion, is refuted by the precise testimony of Aristotle, who declares that the early physicists did not distin­guish the moving cause from the material cause, and that before Hermotimus and Anaxagoras no one postulated a creative intelligence.

It would seem, then, that Thales sought amid the variety of things a single material cause; that he found such a cause in one of the forms of matter most familiar to him, namely, water, and accordingly regarded the world and all that it contains as water variously metamorphosed; and. that he asked himself no questions about the manner of its transformation.

The doctrine of Thales was interpreted and developed in the course of three succeeding generations. First, Anaximander chose for what he called his “ principle ” (àpχή), not water, but a cor­poreal element intermediate between fire and air on the one hand and water and earth on the other. Next, Anaximenes, preferring air, resolved its transformations into processes of thinning and thickening. Lastly, Heraclitus asserted the claims of fire, which he conceived to modify itself, not occasionally, but perpetually. Thus Thales recognized change, but was not careful to explain it; Anaximander attributed to change two directions; Anaximenes conceived the two sorts of change as rarefaction and condensation; Heraclitus, perceiving that, if, as his predecessors had tacitly assumed, change was occasional, the interference of a moving cause was necessary, made change perpetual. But all four agreed in tracing the variety of things to a single material cause, corporeal, endowed with qualities, and capable of self-transformation. A new departure was taken by the Eleatic Parmenides *(q.v.),* who, expressly noting that, when Thales and his successors attributed to the sup­posed element changing qualities, they became pluraliste, required that the superficial variety of nature should be strictly distin­guished from its fundamental unity. Hence, whereas Thales and his successors had confounded the One, the element, and the Many, its modifications, the One and the Not-One or Many became with Parmenides matters for separate investigation. In this way two lines of inquiry originated. On the one hand Empedocles and Anaxagoras, abandoning the pursuit of the One, gave themselves to the scientific study of the Many; on the other Zeno, abandoning the pursuit of the Many, gave himself to the dialectical study oí the One. Both successions were doomed to failure; and the result

*@@@1 Ein mathematisches Handbuch der alten Aegypter* (Leipzig, 1877).

@@@2 Auguste Comte, *Système de Politique Positive,* iii. pp. 297, 300.

@@@3 P. Laffitte, *Les Grands Types de l'Humanité,* vol. ii. p. 292.

*@@@4 Ibid.,* p. 294.

@@@5 Theonis Smyrnaei Platonici *Liber de Astronomia,* ed. Th. H. Martin, p. 324 (Paris, 1849). Cf. Diog. Laër. i. 24.

@@@6 This is the received interpretation of the passage in Diogenes Laertius, i. 24 (see Wolf, *Gesch. der Astron.,* p. 169), where *σϵληvaίov* is probably a scribe’s error for ζωϐιακού. Cf. Apuleius, *Florida,* iv. 18, who attributes to Thales, then old, the discovery: “ quotiens sol magnitudine sua circulum quem permeat metiatur.”

@@@7 In likening the earth to a cylinder Anaximander recognized its circular figure in one direction.

@@@8 See G. V. Schiaparelli, *I Precursorsi di Copernico nell' Antichità,* p. 2 (Milan, 1873).