B in 1904. This work contains an enormous mass of useful work, and gives not only complete technical developments both on the theoretical and practical sides but also has chapters of general interest. The present writer feels it his duty, however, to dissent from Mr Harris’s courageous attempt to construct the cotidal lines of the various oceans.

This work contains the most complete account of the history of tidal theories of which, we know. Laplace’s admirable history of the subject down to his own time has been summarized in § 7. Dr Giovanni Magrini has an appendix to his translation of Darwin’s book, entitled *La Conoscenza della marea nell'antichità,* founded on the researches of Dr Roberto Almagià. Dr Almagià himself gives the results of his researches more fully in a memoir, presented to the Accademia dei Lincei of Rome (5th series, vol. v. fascic. x., 1905, 137 pp).

Another monograph on tides, treating especially the mathematical developments, is Maurice Lévy’s *La Théorie des marées* (Paris, 1898). Colonel Baird’s *Manual of Tidal Observation* (1886) contains instructions for the installation of tide-gauges, and auxiliary tables for harmonic analysis. Airy’s article on "Tides and Waves ” in the *Ency. Metrop.,* although superseded in. many respects, still remains important. Harris’s *Manual* contains a great collection of results of tidal observations made at ports all over the world.

The article “ Die Bewegung der Hydrosphäre ” in the *Encyklopädie der mathematischen Wissenschaften* (vi. I,.1908) gives a technical account of the subject, with copious references. The same article is given in English in vol. iv. (1911) of G. H. Darwin’s collected *Scientific Papers·,* and vols. i. and ii. contain reprints of the several papers by the same author referred to in the present article.

Since the date of the 9th edition of the *Ency. Brit.* some technical discussion of the tides has appeared in textbooks, such as H. Lamb’s *Hydrodynamics.@@1* That work also reproduces in more modern form Airy’s investigation of the effects of friction on the tides of rivers. We are thus able to abridge the present article, but we shall present the extension by Hough of Laplace’s theory of the tides of an ocean- covered planet, which is still only to be found in the original memoirs.

II.—Tide-Generating Forces

§ it. *Investigations of Tide-Generating Potential and Forces.—* We have already given a general explanation of the nature of tide-generating forces; we now proceed to a rigorous investigation. If a planet is attended by a single satellite, the motion of any body relatively to the planet’s surface is found by the process described as reduc­ing the planet’s centre to rest. The planet’s centre will be at rest if every body in the system has impressed on it a velocity equal and opposite to that of the planet’s centre; and this is accomplished by impressing on every body an acceleration equal and opposite to that of the planet’s centre.

Let *M, m* be the masses of the planet and the satellite; *r* the radius vector of the satellite, measured from the planet’s centre; *p* the radius vector,, measured from same point, of the particle whose motion we wish to determine; and *z* the angle between *r* and *p.* The satellite moves in an elliptic orbit about the planet, and the acceleration relatively to the planet’s centre of the satellite is *(M+m)∣P'* towards the planet along the radius vector *r.* Now the centre of inertia of the planet and satellite remains fixed in space, and the centre of the planet describes an orbit round that centre of inertia similar to that described by the satellite round the planet but with linear dimensions reduced in the proportion of *m* to *M-⅛-m.* Hence the acceleration of the planet’s centre is *m∣rt* towards the centre of inertia of the two bodies. Thus, in order to reduce the planet’s centre to rest, we apply to every particle of the system an acceleration m∕r2 parallel to r, and directed from satellite to planet.

Now take a set of rectangular axes fixed in the planet, and let Mι∕, M2r, Mjt be the co-ordinates of the satellite referred thereto; and let .ξp, *ηp, ζp* be the co-ordinates of the particle *P* whose, radius vector is *p.* Then the component accelerations for reducing the planet’s centre to rest are —wM√r2, —wM2∕r2, — *mMi/r\*;* and since these are the differential coefficients with respect to ρξ, p17, pf of the function

and since cos z = Mι^-f-M2η-hMjf, it follows that the potential of the forces by which the planet’s centre is to be reduced to rest is

— ~ cos *z.*

*ri*

Now let us consider the other forces acting on the particle. The planet is spheroidal, and therefore does not attract equally in all directions; but in this investigation we may make abstraction of the ellipticity of the planet and of the ellipticity of the ocean due to the planetary rotation. This, which we set aside, is considered in the theories of gravity and of the figures of planets. Outside its body, then, the planet contributes forces of. which the potential is *M∣p.* Next the direct attraction of the satellite contributes forces of which the potential is the mass of the satellite divided by the distance between the point *P* and the satellite; this is

*m*

√[r3 + pj — *2rp* cos zf

To determine the forces from this potential we regard p and z as the variables for differentiation, and we may add to this potential any constant we please. As we are seeking to find the forces which urge *P* relatively to *M,* we add such a constant as will make the whole potential at the planet’s centre zero, and thus we take as the potential of the forces due to the attraction of the satellite

*m ~ m* √H ÷P2 — 2rρ cos"z} *r ’*

It is obvious that in the case to be considered *r* is very large compared with p,. and. we may therefore expand this in powers of p∕r. This expansion gives us

7i7Λ+7Λ+7Λ+∙∙ψ

where *P∖* = cos *z,* P2 .= f cos2 z — ⅛, *Pt = ξ* cos’ z — ⅞ cosz, &c. The reader familiar with spherical harmonic analysis of course recognizes.the zonal harmonic functions; but the result for a few terms, which is all that is necessary, is easily obtainable by simple algebra.

Now, collecting together the various contributions to the potential, and noticing that y \* -P1 = y^cosz, and is therefore equal and opposite to the potential by which the planet’s centre was reduced to rest, we have as the potential of the forces acting on a particle whose co-ordinates are p∣, p17, pf

y + >⅛ COS3 Z - i) + cos’ z - I COS s) + . . . ( I)

The first term of (1) is the potential of gravity, and the terms of the series, of which two only are written, constitute the ea · tide-generating potential. In all practical applications this series converges so rapidly that the first term is amply suffi­cient, and thus we shall generally denote

(cos’z- ∣) (2)

as the tide-generating potential.@@2

At the surface of the earth p is equal to *a* the earth’s radius.

§ 12. *Form of Equilibrium.—*Consider the shape assumed by an ocean of density *σ, on* a planet of mass *Mt* density δ and radius *a,* when acted on by disturbing forces whose potential is a solid spherical harmonic of degree *it* the planet not being in rotation.

If St∙ denotes a surface spherical harmonic of order *i,* such a potential is given at the point whose radius vector is p by

r=^(≡)^∙ ω

In the case considered in § 11, 1=2 and S< becomes the second zonal harmonic cos2 *z~j.*

The theory of harmonic analysis tells us that the form of the ocean, when in equilibrium, must be given by the equation

P = α+e\*Si. (4)

Our problem is to evaluate e\*. We know that the external potential of a layer of matter, of depth e,Si and density σ, has the value

Hence the whole potential externally to the planet and up to its surface is

f+W-s∙+⅛τ⅜(Γ,^ “>

The first and most important term is the potential of the planet, the second that of the disturbing force, and the third that of the departure from sphericity.

Since the ocean must stand in a level surface, the expression (5) equated to a constant must be another form of (4). Hence, if we put ρ=α+e<S< in the first term of (5) and p=α in the second and third terms, (5) must be constant; this can only be the case if the

.@@@1 The theory as presented in the *Mécanique céleste* is unnecessarily difficult, and was much criticized by Airy. Before the publication of the 9th and 10th editions of the *Ency. Brit,* it was necessary for the student to read a number of controversial papers published all over the world in order to get at the matter.

@@@2 The reader may refer to Thomson and Tait’s *Natural Philosophy* (1883), pt. ii. §§ 798-821, for further considerations on this and analogous subjects, together with some interesting examples.