Since these equations must be true identically, the coefficients of PJ in each of them must vanish. Therefore α'i<r-i(f+l)∣ + 2σ\* j ‰¾y+tf-⅛Ξ⅞ j =0.

—Ä(<r+«(» + l))+^^CJ = O,

w-w,Hfc^⅛a (1S

+⅛(ι+i+l¾¾2,tl'+jll-⅛l-o∙

If we eliminate the α,s and *β,s* from the third equation (45), by means of the first two, we find

*^C↑-i-L∙Cti+η∙^≈^E∙,* (46)

where

,. \*' ■ (P-s\*)k+(f-ι)(f-2)]

L< '°σi[σ+>(i + t)l+ (4? -1) [σ- (í — I)«][σ+i(ι +1))

, [(f+ι)a-⅜+(<+2)(t+3)l t⅞<

+[4(i + l),-Ij[σ-(l + l)(\*+2)][<r+i(f+l>l *4ma* -(t→)(i-S-l)

ii", (2Í - l)(2r-3)[σ- (t- l)lj'

• -(\*'+\*+I)(\*'+\*+2)

r,i+1 (2t+3) (2» +5) [0— (»+1) («+2)]·

In the case of the luni-solar semi-diurnal tide (called K2 in the notation of harmonic analysis) we have *i=2, s = 2, σ=2.* Hence it would appear that these formulae for £J and £J\_, fail by becoming indeterminate, but i and s are rigorously integers, whereas σ depends on the “ speed” of the tide; accordingly in the case referred to we must regard terms involving (í — *s)* as vanishing in the limit when σ approaches to equality with i (t — 1). For this particular case then we find

⅛-r⅛-li'-

The equation (46) for the successive *C's* is available for all values of » provided that C-1, E-ι, Co, £0 are regarded as being zero.

As in the case of the zonal oscillations, the equations with odd suffixes separate themselves from those with even suffixes, so that the two series may be treated independently of one another. In­deed, as we shall see immediately, the series with odd suffixes are satisfied by putting all the C’s with odd suffixes zero for the case of such oscillations as may be generated by the attractions of the moon or sun.

For the semi-diurnal tides « = 2, r=2,' and ∕ is approximately equal to unity. Hence the equilibrium tide is such that all the *E‘,* excepting *E%,* are zero.

For the diurnal tides t=2, s = I, and *f* is approximately equal to ⅛. Hence all the *E\*,* excepting *Ej,* are zero. Since in neither case is there any *E* with an odd suffix, we need only consider those with even suffixes. ,

In both cases the first equation among the C’s is -XiCi+√iCj≡⅛E∙,(i=2or1). It follows that if we write

^=-è£î(î±=20rl)· the equation of condition amongst the C's would be of general appli­cability for all even values of í from 2 upwards.

The symbols £·o, ιft do not occur in any of the equations, and therefore we may arbitrarily define them as denoting unity, although the general formulae for *ζ* and *η* would give them other values. Accordingly we shall take

^=c0=-⅛^(i = 2or 1>∙

With this definition the equation

*‰-tCi-t~LiC[∙⅛∙ηi+2Cl+t = θ(,S≈2* or 1)

is applicable for *i~2,* 4, 6, &c.

It may be proved as in the case of the tides of long period that we may regard *C,i[C↑+t* as tending to zero. Then our equation may be written in the form

m Q~1≈L,- i⅛ι+S

*~ C↑ ', W∕C⅛,*

and by successive applications the right-hand side may be expressed in the form of a continued fraction. Let us write

p,=f⅛' i⅛'-n ‰⅛h l;- *L!+,- ‰-∙∙∙* Hence our equation may be written

ξi-tTj^ ff∙i

Whence C∙≡-C∙ .

*, ΊΪ ,~t*

It follows that

C∙ = ~C∙, C·=—1—C∙, C∙ = -*——-C∙,* Ac.

, ¾∙ »’ < »’ · ⅛⅛ ∙'

Then since we have defined

,j∙=ιandCJ=-i⅛,

all. the C’s are expressed in terms of known quantities. Hence the height of tide b is given by

b = ∑⅛j c0s(2n∕i+s≠+α)

= ∞≡(2^+\* + >) · ∙]

But the equilibrium tide c is given by

c = EJPJ c0s(2n∕t-f-s<⅛+a).

Hence we may write our result in the following form, which shows the relationship between the true dynamical tide and the equilibrium tide :—

. γ⅛ t ( rrtn, , g{Hιτ,, , )

*4ma P[ Γ1'p'+ η↑ p«+* p<+∙∙∙5

From a formula equivalent to this Mr Hough finds for the lunar semi-diurnal tide (s = 2), for a sea of 1210 fathoms =— ),

∖4mu 40/ b=⅛) ∙10396-P2 + ∙57998^i-∙i9273^i+∙03θ5ψPΓ∙∙ ∣ · This formula shows us that at the equator the tide is “ inverted,” and has 2∙4187 times as great a range as the equilibrium tide.

For this same ocean he finds that the solar semi-diurnal tide is " direct ” at the equator, and has a range 7∙9548 as great as the equilibrium tide.

Now the lunar equilibrium tide is 2∙2 times as great as the’solar equilibrium tide, and since 2∙2×2∙4187 is only 5∙3, it follows that in such an ocean the solar tides would have a range half as great again as the lunar. Further, since the lunar tides are " inverted ” and the solar “ direct," spring tide would occur at quarter m∞n and neap tide at full and change.

We give one more example from amongst those computed by Mr Hough. In an ocean of 9680 fathoms (γ∕4nια = ι∕5), he finds b=p∣ ∣ 1∙7646PJ- ∙o6o57PJ!+∙ooi447PJ... j .

At the equator the tides are “ direct ” and have a range of 1-9225 as great as the equilibrium tide. In this case the tides approximate in type to those of the equilibrium theory, although at the equator, at least, they have nearly twice the range.

We do not give any numerical results for the diurnal tides, tor reasons which will appear from the following section.

§ 19. *Diurnal Tide approximately [evanescent.—*The equilibrium diurnal tide is given by

*c≈Ei Pì* cos(2n∕i-⅛-≠4-α), where *f* is approximately ⅛ and the associated function for » = 2, s = ι is

P⅛ =3 sin *θ* cos *θ.*

Now the height of tide is given by

b=∑CJPJ c0s(2ny∕+φ+α), and the problem is to evaluate the constants *C‘.*

If possible suppose that b is also expressed by a single term like that which represents c, so that

b = 3C⅛ sin *θ* cos *6* c0s(2n∕∕+φ+α).

Then the differential equation (22) to be satisfied becomes

ì *∕ ■ adu 1* I .∖ cos *β du , u* λ

d (s1π⅛÷rc0sa) -~7- aF+≡⅞

Υt , \*'(sιnβ<70∖ *f2-*cos20 ∕ sinβ(∕2-cos2fl) )

+4nιι1 *C⅛u≈o,* where *u* is written for brevity in place of sin *θ* cos *θ.*

Now when/is rigorously equal to ∣, it may be proved by actual differentiation that the expression inside the brackets { ) vanishes

identically, and the equation reduces to Cl =0.

We thus find that in this case the differential equation is satisfied by zero oscillation of water-level. I n other words we reach Laplace's remarkable conclusion that there is no diurnal rise and fall of the tides. There are, it is true, diurnal tidal currents, but they are so arranged that the water level remains unchanged.

In reality/is not rigorously ⅛ (except for the tide called *Kt)* and there will be a small diurnal tide. The lunar diurnal tide called 0 has been evaluated for various depths of ocean by Mr Hough and is found always to be small.

§ 20. *Free Oscillations of the Ocean.—*Mr Hough discusses the various types of free oscillations of the ocean. They are very com­plex, and consist of westward waves and eastward waves of very various periods. He finds, as was to be expected, that if, for an ocean of given depth, a free wave very nearly coincides in period