arise from the potential *V≈∙^pi(i* cos\* 2 —J cos ≈). They give rise to a small diurnal tide Mi, and to a small ter-diurnal tide M2; but we shall not give the analytical development.

§ 26. *Over-Tides. Compound Tides ana Meteorological Tides.— We* have in § 23 stated results derived from dynamical theory as to over-tides, which represent the change of profile of the wave as it advances in shallow water. The only tides in which it has hitherto been thought necessary to represent this change of form belong to the principal lunar and principal solar series. Thus, besides the fundamental astronomical tides M2 and S2, the over-tides Ml, Me, Me and S4, Sβ are usually deduced by harmonic analysis.

Compound tides have been also referred to in § 23; they represent a result of the combination of two waves of different speeds travel­ling through shallow water. On combining the speeds of the important tides, it will be .found that there is in many cases a compound tide which has itself a speed identical with that of an astronomical or meteorological tide. We thus find that the tides O, Kn P, M2, Mf, Q, Mi, L are liable to perturbation in shallow water. We refer to the *Brit. Assoc. Report* for 1883 or to Harris’s *Manual* for a schedule, with initials, of the compound tides which are usually evaluated.

All tides whose period is an exact multiple or submultiple of a mean solar day, or of a tropical year, are affected by meteorological conditions. Thus all the tides of the principal solar astronomical series S, with speeds 7 —*η,* 2(7—Ό, 3(υ^^ι)∙ &c., are subject to more or less meteorological perturbation. An annual inequality in the diurnal meteorological tide ⅛ will also give rise to a tide *y-2τ∣,* and this will be fused with and indistinguishable from the astronomical P; it will also give rise to a tide with speed 7, which will be in­distinguishable from the astronomical part of Ki. Similarly the astronomical tide K2 may be perturbed by a semi-annual inequality in the semi-diurnal astronomical tide of speed 2(7—1;). Although the diurnal tide Si or *y—η* and the semi-annual and annual tides of speeds *2η* and *η* are all quite insensible as arising from astronomical causes, yet they have been found of sufficient importance to be con­sidered. The annual and semi-annual tides are of enormous importance in some rivers, representing in fact the yearly flooding in the rainy season. In the reduction of these tides the arguments of the S series are *t, 2l,* 3/, &c., and of the annual, semi-annual, ter-annual tides *h, 2h,* 3A. As far as can be foreseen, the magni­tudes of these tides are constant from year to year.

§ 27. *On the Form of Presentation of Results of Tidal Observations.—* Supposing *n* to be the speed of any tide in degrees per mean solar hour, and *t* to be mean solar time elapsing since <Λ of the first day of (say) a year of continuous observation, then the immediate result of harmonic analysis is to obtain a height R and an angle f such that the height of this tide at the time *I* is given by

R cos (nt—f).

R is the semi-range of the tide (say) in British feet, and f is an angle such that f/n is the time elapsing after oh of the first day until it is high water of this particular tide. It is obvious that f may have any value from 0o to 3600, and that the results of the analysis of successive years of observation will not be comparable with one another when presented in this form.

But let us suppose that the results of the analysis are presented in a number of terms of the form

fH cos *(V+u-\*),*

where *V* is a linear function of the moon’s and sun’s mean longi­tudes, the mean longitude of the moon’s and sun’s perigees, and the local mean solar time at the place of observation, reduced to angle at 150 per . hour. *V* increases uni­formly with the time, and its rate of increase per mean solar hour is the *n* of the first method, and is called the speed of the tide. It is supposed that *u* stands for a certain function of the longitude of the node of the lunar orbit at an epoch half a year later than oh of the first day. Strictly speaking, *u* should be taken as the same function of the longitude of the moon’s node, varying as the node moves; but, as the varia­tion is but small in the course of a year, *u* may be treated as a constant and put equal to an average value for the year, which average value is taken as the true value of *u* at exactly mid year. Together *Vfi-u* constitute that function which has been tabulated as the "argument” in the schedules of §25. Since *V-f-u* are together the whole argument according to the equilibrium theory of tides,, with sea covering the whole earth, it follows that √n is the lagging of the tide which arises from kinetic action, friction, of the water, imperfect elasticity of the earth, and the distribution of land It is supposed that H is the mean value in British feet of the semi-range of the particular tide in question; f is a numerical factor of augmentation or diminution, due to the variability of the obliquity of the lunar orbit. The value of f is the ratio of the ” coefficient ” in the third column of the preceding schedules to the mean value of the same term. It is obvious, then, that, if the tidal observations are consistent from year to year, H and *κ* should come out the same from each year's reductions. It is only when the results are presented in such a form as this that it will be possible to judge whether the harmonic analysis is yielding satisfactory results. This mode of giving the tidal results is also essential for the use of a tide-predicting machine (see § 8).

We must now show how to determine H and *κ* from R and J^. It is clear that H≈R∕f, and the determination of f from the schedules depends on the evaluation of the mean value of each of the terms in the schedules, into which we shall not enter. If ½1 be the value of *V* at oh of the first day when *t* is zero, then clearly

— f — Vo+m—\*, so that \*=f+Ro+w-

Thus the rule for the determination of *κ* is: *Add to the value of ζ the value of the argument at of, of the first day.*

The results of. harmonic analysis are usually tabulated by giving Η, *κ* under the initial letter of each tide; the results are thus com­parable from year to year.@@1 For the purpose of using the tide-predicting machine the process of determining H and κ from R and f has simply to be reversed, with the difference that the instant of time to which to refer the argument is oh of the first day of the new year, and we must take note of the different values of *u* and f for the new year. Tables’ have been computed for f and *u* for all longitudes of the moon’s node and for each kind of tide, and the mean longitudes of moon, sun, and lunar perigee may be extracted from any ephemeris. Thus when the mean semi-range II and the retardation *κ* of any tide are known its height may be computed for any instant. The sum of the heights for all the principal tides of course gives the actual height of water.

§ 28. *Numerical Harmonic Analysis.—*The tide-gauge furnishes us with a continuous graphical record of the height of the water above some known datum mark for every instant of time. The first operation performed on the tidal record is the measurement in feet and decimals of the height of water above the datum at every mean solar hour. The period chosen for analysis is about one year and the first measurement corresponds to noon.

If T be the period of any one of the diurnal tides, or the double period of any one of the semi-diurnal tides, it approximates more or less nearly to 24 solar hours, and, if we divide it into 24 equal parts, we may speak of each as a T-hour.

The process of harmonic analysis consists of finding the average height of water at each of the 24 T-hours of the T-day, but we shall not go into the way in which this may be done.@@2 It must suffice to say that it depends on the fact that in the long ruħ any given T-hour will fall at all hours of any other special day.

The final outcome is that we obtain the height of water at each of the 24 T-hours of a T-day, freed from the influence of all the other tides. We may see that it is thus possible to isolate the T-tide. When this has been done let *t* denote T-time expressed in T-hours, and let *n* be 15°. Then we express the height *h* as given by the averaging process above indicated by the formula

⅛ = A0 + A1 cos πt4-Bι sin n∕⅛A2 cos 2wi + B2 sin 2n∕+ . . ., where *t* is o, 1, 2,. . . 23.

See the article Harmonic Analysis for the numerical processes by which Ao. Aj, Bi, A2, B2, &c·, may be evaluated. It is obvious that such a formula as A cos nt-∣-B sin *nt* may easily be reduced to the form R cos *(nt-ξ}. .* An actual numerical example of harmonic analysis of tidal observations is given in the *Admiralty Scientific Manual* (1886) in the article "Tides,” or G. H. Darwin’s *Scientific Papers,* vol. i.@@3

V.—Synthetic Method

§ 29. *On the Method and Notation.—*The general nature of the synthetic method has been already explained; we now propose to show how the expressions for the tide may be developed from the result as expressed in the harmonic notation. If it should be desired to make a comparison of the results of tidal observation as expressed in the synthetic method with those of the harmonic method, or the converse, or to compute a tide-table from the har­monic constants by reference to the moon’s transits and the declina­tions and parallaxes of sun and moon, analytical expressions founded on a procedure indicated in the following sections are necessary.

In chapter iv. the mean semi-range and angle of retardation of any one of the tides have been denoted by H and *κ.* We shall here, however, require to introduce several of the H's and κ's. into the same expression, and they must therefore be distinguished from one another. This may in general be done conveniently, by writing as a subscript letter the initial of the corresponding tide; for example Hm, κm will be taken to denote the H and *κ* of the lunar tide M1. This notation does not suit the K2 and Ki tides,

@@@1 See, for example, a collection of results by Baird and Darwin, *Proc. Roy. Soc.* (1885), No. 239, and a more extensive one in Harris’s ilfαnuol.

*@@@2 Report on Harmonic Analysis to Brit. Assoc.* (1883), and more extended table in Baird's *Manual of Tidal Observation* (London, 1887).

@@@3 See Darwin’s *Tides* for an account without mathematics.