and we shall therefore write H\*, κ\* for the semi-diurnal K2, and H', √ for the diurnal K1 tide. These two tides proceed according to sidereal time and arise from the sun and moon jointly, and a synthesis of the two parts of each is effected in the harmonic method, although that synthesis is not explained in chapter iy. It is now necessary to reverse this partial synthesis, in order to obtain a more complete one. We must therefore note that the ratio of the solar to the lunar part of the total K2 tide is 0\*46407; so that 0\*683 H\* is the lunar portion of the total K2. There will be no occasion to separate the two portions of Ki, and we shall retain the synthesis which is effected in the harmonic method.

§ 30. *Semi-Diurnal Tides.—*The process adopted is to replace the mean longitudes and elements of the orbit in each term of the harmonic development of the schedules of § 25 by hour-angles, declinations, and parallaxes.

At the time *I* (mean solar time of port reduced to angle) let α, δ, *ψ* be d’s R.A., declination, and hour- angle, and *I* d’s longitude measured from the “ inter­section.” These ana other symbols when written with a subscript accent are to apply to the sun. Then *v* being the R.A. of the intersection, we have from the right- angled spherical triangle of which the sides are Z, δ, *a — v* the relations tan (α—p) ≡≡ cos *I* tan Z, sin δ ≡ sin *I* sin Z. (49)

Now s—ξ is the d’s mean longitude measured from the intersection and *s—p* is the mean anomaly; hence approximately

*I* = s *— ξ + 2e* sin (s — p). (50)

From (49) and (50) we have approximately

α = s + *(v* — ξ) + *2e* sin (5 — *p)* —tan2 ⅛I sin 2(5 — ξ).

Now, Ã being the O ’s mean longitude, *t + h* is the sidereal hour- angle, and *ψ = t + h — a.*

Hence

*t-∖-h-s— (v—£) ≈ψ-∖-2e* sin (5—*p)* — tan2 ⅛ *I* sin 2(5—ξ). » (51) Again, if we put

cos2∆ = I — ⅛ sin2 *I* (52)

we have approximately from (49) and (50)

cos2δ — cos2∆ z fc. 1

-.-≈≡--~a<≈-ξ> . (.,) whence sin δ cos δ *dδ Ç*

*σ* sin3∆ *dt ( ) )*

Obviously Δ is such a declination that sin2 Δ is the mean value of sin2 δ during a lunar month. Again, if.P be the ratio of the. d’s parallax to her mean parallax, the equation to the ellipse described gives ■“(-? — 0 — cos (5 “\* *P} I*

*u* 1 dP . z f, ⅛4)

whence - — = sin (5 — *p) ∖*

*e(σ -<S)dt ' r'*

Now it appears in schedule A of § 25 that the arguments of all the lunar semi-diurnal tides are of the form 2(∕-∣-A-*v) ±* 2(s—ξ) or =fc(s-£). It is clear, therefore, that the cosines of such angles may by the relations (51), (53), (54) be expressed in terms of hour-angles, declinations and parallaxes. Also.by means of (52) we may intro­duce Δ in place of *I* in the coefficients of each term. An approxi­mate formula for Δ is 16∙510-}-3∙440 cos Λr-0∙19o cos 2 AC Details will be found in the *Brit. Assoc. Report* for 1885.

We shall not follow the analytical processes in detail, but the formulae given show the possibility of re placing, the symbols used in the methodOf harmonic analysis by others involving R.A., declination and parallax..

Before giving the results of the processes indicated it must be remarked that greater succinctness is obtained by the introduction of the symbol δ' to denote the d’s declination at a time earlier than that of observation by an interval which may be called the “ age of the declinational inequality,” and is computed from, the formula tan (κ\*-κm)∕2σ or 52∙2h tan (κ — κm). Similarly, it is convenient to introduce *P,* to denote the value of *P* at a time earlier than that of observation by the ti age of the parallactic inequality,” to be computed from tan (κm — κn)∕(σ-©) or ιo5\*3h tan (κm — κn). These two “ ages ” probably do not differ in general much from a third period, computed from (κ1-κm)∕2(σ-η), which is called the “ age of the tide.”

. The similar series of transformations when applied to the solar tides lead to simpler results, because ∆z is a constant, being 16∙330,.and the “ ages ” may be treated as zero; besides the terms depending on *dδi∣dt* and *dPf∣dt* are negligible. Formulae for the semi-diurnal tide, of great exactness are obtainable by means of these transformations, but they lack the simplicity of those obtained in the harmonic method.. On the other hand they are in some respects even more exact, since all lunar inequalities are represented. We shall not give the complex formulae which represent the com­plete substitution of R.A., declination and parallax in the earlier formulae, but shall content ourselves with rougher results, which are still fairly accurate.

Let us write them

*∙\* ∣f* TJ I COS2 δ “\* COS2 Δ τj m *tn ∖*

m =1 ⅛ιu + —≡ι≡≡τ≡7—°∙6i⅛ h' ∞≡ (-≈' - <⅛),

1 cos2δ'-cos2∆ e.a H\* . t v .

2μ ≡ κm + ¾—°’683 Hisιn <κ “ l(55)

M = P ¾ cos\*⅜. H,

m' ,' cos2∆, t,

*2μt ≈κt.*

Then we find that the height h2 of the complete lunar and solar semi­diurnal tide is represented with a fair degree of approximation by

h2 = M cos 2 (≠ — μ).-J- Mz cos 2(≠z — μ,). (56)

The first of these is the lunar tide, and it will be observed that the height M depends on the cube of the moon’s parallax at a time earlier than that of observation by “ the age of the parallactic? inequality,” and that it depends also on the moon’s declination at a time earlier by “ the age of the declinational inequality.” The phase, of. the tide, represented by the angle *2μ,* also has ajdeclinational inequality.

The second, term is the solar tide, and it also has parallactic and declinational inequalities.

The formulae (55), (56) have been used in an example of the computation of a tide-table given in the *Admiralty Scientific Manual* (1886).

§ 31. *Synthesis of Lunar and Solar Semi-Diurnal Tides.—*Let A be the excess of d’s over Q’s R.A., so that

A =α-αy, )

≠y = ≠ + A, [ (57)

and h2 = M cos 2(≠-μ)+M, cos 2(≠+A-μ,). )

The synthesis is then completed by writing

H COS2(μ-*ψ)* = M ÷M, cos 2(A—μy⅛μ), H sin 2(µ—φ)= My sin 2(A-μ,-j-μ), so that h2=≡H cos 2(≠-≠). (58)

Then H is the height of the total semi-diurnal tide and φ∕(-y-dα∕d∕) or approximately .≠∕(γ-σ) or 3s⅛ *φ,* when *φ* is given in degrees, is the ft interval ” from the moon’s transit to high water.

The formulae for H and *φ* may be written

H = √{M2 + M,2+2MMzCOS2(A-μ,÷μ)∣ ) ta∏2(u-<⅛)= M, sin 2 (A-μ,÷μ) I (59)

ta∏2U\* *Φ)* M+Mi∞S2(A-μ,+μ) )

Since A goes through its period in a lunation, it follows that H and *Φ* have inequalities with a period of half a lunation. These are called the u fortnightly inequalities in the height and interval.

Spring tide obviously. occurs when A==μz-μ. Since the mean value of A is *s—h* (the difference of the mean longitudes), and since the mean values of µ and *µ,* are Jκm, ⅜κβ, it follows that the mean value of the period, elapsing after full moon and change of moon up to spring tide is (κtyκm)∕2.(σ-*η).* The association of spring tide with full and change is obvious, and a fiction has been adopted by which it is held that spring tide is generated in those con­figurations of the m∞n and sun, but takes some time to reach the port of observation. Accordingly *(κt-κm)∣2(σ-η)* has been called the u age of the tide.”. The average age is about 36 hours as far as observations have yet been made. The age of the tide appears not in general to differ very much from the ages of the declinational and parallactic inequalities.

In computing a tide-table it is found practically convenient not to use A, which is the difference of R.A.is at the unknown time of high-water, but to refer the tide to. Ao, the difference of R.A.’s at the time of the moon’s transit. It is clear that Ao is the apparent time of the moon’s transit reduced to angle at 150 per hour. Wc have already remarked that *φ∣(y-da∣dt)* is the interval from transit to high-water, and hence at high-water

λ λ 1 dα∕<∕Z-dα,∕dZ j .

*A=A»+ y-da∕<-φ-* (60)

As an approximation .we may attribute to all the quantities in the second term their mean values, and we then have

A = Ao+^m

and A — μ,+ µ =≡ Ao— *µ, ÷* Ao — µ, + ijμ. (61)

This approximate formula (61) may be used in computing from (59) the fortnightly inequality in the “ height ” and “interval.”.

In this investigation we have supposed that the declinational and parallactic corrections are applied to the lunar and solar tides before their synthesis; but it is obvious that the.process may.be reversed, and that we may form a table of the fortnightly inequality based on mean values Hm and Hj and afterwards apply corrections. This is the process usually adopted, but it is less exact. The labour of computing the fortnightly inequality, especially by graphical methods, is not great, and the plan here suggested seems preferable.

§ 32. *Diurnal Tides.*—These tides have not been usually treated