with much completeness in the synthetic method. In the tide­tables of the British Admiralty we find that the tides at some ports are “affected by diurnal inequality”; such a statement may be interpreted as meaning that the tides are not to be predicted by the information given in the so-called tide-table. The diurnal tides are indeed complex, and do not lend themselves easily to a complete synthesis. In the harmonic notation the three important tides are ⅛, O1 P1 and the lunar portion of Ki is nearly equal to 0 in height, whilst the solar portion is nearly equal to P. A complete synthesis may be carried out on the lines adopted in treating the semi-diurnal tides, but the advantage of the plan is lost in conse­quence of large oscillations of the amplitude through the value zero, so that the tide is often represented by a negative quantity multiplied by a circular function. It is best, then, only to attempt a partial syn­thesis, and to admit the existence of two diurnal tides. One of these will be a tide consisting of K1 and P united, and the other will be O.

We shall not give the requisite formulae, but refer the reader to the *Bril. Assoc. Report* for 1885. A numerical example is given in the *Admiralty Manual* for 1886.

§ 33. *On the Reduction of Observations of High- and Low-Water.@@1 -* A continuous register of the tide or observation at fixed intervals of time, such as each hour, is certainly the best ; but for the adequate use of such a record some plan analogous to harmonic analysis is necessary. Observations of high- and low-water only have, at least until recently, been more usual. In the reduction the immediate object is to connect the times and heights of high- and low-water with the moon’s transits by means of the establishment, age and fortnightly inequality in the interval and height. The reference of the tide to the establishment is not, however, scientifically desirable, and it is better to determine the mean establishment, which is the mean interval from the moon’s transit to high-water at spring tide, and the age of the tide, which is the mean period from full moon and change of moon to spring tide.

For these purposes the observations may be conveniently treated graphically.@@2 An equally divided horizontal scale is taken to represent the twelve hours of the clock of civil time, regulated to the time of the port, or—more accurately— arranged always to show apparent time by being fast or slow by the equation of time; this time-scale represents the time-of-clock of the moon's transit, either upper or lower. The scale is perhaps most conveniently arranged in the order V, VI,...X1I, I...IIII. Then each interval of time from transit to high-water is set off as an ordinate above the corre­sponding time-of-clock of the moon’s transit. A sweeping curve is drawn nearly through the. tops of the ordinates, so as to cut off minor irregularities. Next along the same ordinates are set off lengths corresponding to the height of water at each high-water. A second similar figure may be made for the interval and height at low-water. In the curve of high-water intervals the ordinate corresponding to XII. is the establishment, since it gives the time of high-water at full moon and change of moon. That ordinate of high-water intervals which is coincident with the greatest ordinate of high-water heights gives the mean establishment. Since the nιoon⅛ transit falls about fifty minutes later on each day, in setting off a fortnight’s observations there will be about five days for each four times-of-clock of the upper transit. Hence in these figures we may regard each division of the time-scale I to H, II to III, &c·, as representing twenty-five hours instead of one hour. Then the distance from the greatest ordinate of high-water heights to XII is called the age of the tide. From these two figures the times and heights of high- and low-water may in general be predicted with fair approximation. We find the time-of-clock of the moon's upper or lower transit on the day, correct by the equation of time, read off the corresponding heights of high- and low-water from the figures, and the intervals being also read off are added to the time of the moon’s transit and give the times of high- and low-water. At all ports there is, however, an irregularity of heights and intervals between successive tides, and in consequence of this the curves present more or less of a zigzag appearance. Where the zigzag is perceptible to the eye, the curves must be smoothed by drawing them so as to bisect the zigzags, because these diurnal inequalities will not present themselves similarly in the future. When, as in some equatorial ports, the diurnal tides are large, this method of tidal prediction fails in the simple form explained above. It may however be rendered applicable by greater elaboration.@@3

This method of working out observations of high- and low-water was not the earliest. In the *Mécanique Céleste,* bks. i. and v., Laplace treats a large mass of tidal observations by dividing them into classes depending on the configurations of the tide-generating bodies. Thus he separates the two syzygial tides at full moon and change of moon and divides them into equinoctial and solstitial tides. He takes into consideration the tides of several days

embracing these configurations. He goes through the tides at quadratures on the same general plan. .The effects of declination and parallax and the diurnal inequalities are similarly treated. Lubbock *(Phil. Trans.,* 1831, seq.) improved the method of Laplace by taking into account all the ob­served tides, and not merely those appertaining to certain configurations. He divided the observations into a num­ber of classes. First, the tides are separated into parcels, one for each month ; then each parcel is sorted according to the hour of the moon’s transit. Another classification is made according to declination; another according to parallax; and a last for the diurnal inequalities. This plan was followed in treating the tides of London, Brest, St Helena, Plymouth, Portsmouth and Sheemess. Whewell *(Phil. Trans.,* 1834, seq.) did much to reduce Lubbock’s results to a mathe­matical form, and made a highly important advance by the intro­duction of graphical methods by means of curves. The method explained above is due to him. Airy remarks of Whewell’s papers that they appear to be “ the best specimens of reduction of new observations that we have ever seen.’

VI.—Tidal Deformation of the Solid Earth

§ 34. *Elastic Tides.—*The tide-generating potential varies as the square of the distance from the earth’s centre, and the correspond­ing forces act at every point throughout its mass. No solid matter possesses the property of absolute rigidity, and φwe must therefore admit the probable existence of tidal elastic deformation of the solid earth. The problem of finding the state of strain of an elastic sphere under given stresses was first solved by G. Lamé;@@4 he made, however, but few physical deductions from his solution. An independent solution was found by Lord Kelvin,@@5 who drew some interesting conclusions concerning the earth.

His problem, in as far as it is now material, is as follows. Let a sphere of radius *a* and density *w* be made of elastic material whose bulk and rigidity moduli are *k* and n, and let it be subjected to forces due to a potential per unit volume, equal to τwr\*(sin\*λ-i), where λ is latitude. Then it is required to find the strain of the sphere. We refer the reader to the original sources for the methods of solution applicable to spherical shells and to solid spheres. The investigation applies either to tidal or to rotational stresses. In the case of tides τ = Jw∕c∙, *m* and *c* being the moon’s mass and distance, and in the case of rotation τ= — jωs, ω being the angular velocity about the polar axis. The equation to the surface is found

~∙l'∙+⅛['÷⅛⅛H->i∙

In most solids the bulk modulus is considerably larger than the rididity modulus, and in this discussion it is sufficient to neglect *n* compared with *k.* With this approximation, the ellipticity *e* of the surface becomes

*5wat*

*e =*  τ.

19n

Now suppose the sphere to be endued with the power of gravitation, and write

r= ⅛ 9=μ,

*ζwa1* ≈ 5 *a*

where g is gravity at the surface of the globe. Then, if there were no elasticity, the ellipticity would be given by e = τ∕g, and without gravitation by *e=τ∣τ.* And it may be proved in several ways that, gravity and elasticity co-operating,

*τ \_ τ* I e r+s-3'ι + r∕g'

If *n* be the rigidity of steel, and if the globe have the size and mean density of the earth, r∕<j = 2, and with the rigidity of glass r∕<j = j. Hence the ellipticity of an earth of steel under tide-generating force would be ⅛ of that of a fluid earth, and the fraction for glass would be ⅛. If an ocean be superposed on the globe, the visible tide will be the excess of the fluid tide above the solid tide. Hence for steel the oceanic tides would be reduced to j, and for glass to J of the tides on a rigid earth.

It is not possible in general to compute the tides of an ocean lying on an unyielding nucleus. But Laplace argued that friction would cause the tides of long period (§ 17) to conform to the equilibrium law, and thus be amenable to calculation. Acting on this belief, G. H. Darwin discussed the tides of long period as observed during 33 years at various ports, and found them to be ∣ as great as on an unyielding globe, indicating an elasticity equal to that of steel.@@6· Subsequently W. Schweydar repeated the calculation from 194 years of observation with nearly the same result.@@7 But as Laplace’s argument appears to be unsound (§ 17), the conclusion seems to become of doubtful validity. Yet subsequently Lord Rayleigh showed

@@@1 Founded on Whewell’s article “ Tides,” in *Admiralty Manual* (ed. 1841), and on Airy’s " Tides and Waves,” in *Ency. Metrop.*

@@@2 For a numerical treatment, see *Directions for Reducing Tidal Observations,* by Commander Burdwood, R.N. (London, 1876).

@@@3 G. H. Darwin " On Tidal Prediction,” *Phil. Trans.* (1891), vol. 189 A.

*@@@4 Théorie math, de l'élasticité* (1866), p. 213.

@@@5 Thomson and Tait, *Nat. Phil.* §§ 732-737 and 833-842, or *Phil. Trans.* (1863), pt. ii., p. 583. Compare, however, J. H. Jeans, *Phil. Trans.* (1903), 201 A, ρ. 157.

@@@6 Thomson & Tait, *Nat. Phil.* § 843.

*@@@7 Beiträge zur Geophysik* (1907) ix. 41.