with the axis of orbital momentum. When, *x* is very small, the equation becomes F=—ι∕x,. Hence the axis of Y is asymptotic on both sides to the curve of energy. If the line of momentum intersects the curve of rigidity, the curve of energy has a maximum vertically underneath the point of, intersection nearer the origin and a minimum underneath the point more remote. But if there are no intersections, it has no maximum or minimum.

Fig. 8 shows these curves when drawn to scale for the case of the earth and moon, that is to say, with /1=4. The points *a* and *bt* which are the maximum and minimum of the curve of energy, are supposed to be on the same ordinates as Λ and B, the intersections of the curve of rigidity with the line of momentum. The intersection of the line of momentum with the axis of orbital momentum is

denoted by D, but in a figure of this size it necessarily remains indistinguishable from B. As the zero of energy is quite arbitrary the origin for the energy curve is displaced downwards, and this prevents the two curves from crossing,one another in a confusing manner. On account of the limitation imposed we neglect the case where the quartic has no real roots. Every point of the line of momentum gives by its abscissa and ordinate the square root of the satellite’s distance and the rotation of the planet, and the ordinate of the energy curve gives the energy corresponding, to each distance of the satellite. Part of the figure has no physical meaning, for it is impossible for the satellite to move round the planet at a distance less than the sum of the radii of the planet and satellite. For example, the moon’s diameter being about 2200 m. and the earth’s about 8000, the moon’s distance cannot be less than 5100 miles. Accordingly a strip is marked off and shaded, on each side of the vertical axis within which the figure has no physical meaning. The point *P* indicates,the, present configuration of the earth and moon. The curve of rigidity *x3y* = 1 is tne same for all values of *h,* and by moving the line of momentum parallel to itself nearer to or further from the origin, we may represent all possible moments of momentum of the whole system. The smallest amount of moment of momentum with

1 which it is possible to set the system moving as a rigid body, with centrifugal force enough to balance the mutual attraction, is when the line of momentum touches the curve of rigidity. The condition for this is clearly that the equation x4-½x8⅛ι=o should have equal roots. If it has equal roots, each root must be and therefore

d⅛p-⅛(i⅛r+ι≈o,

whence Λ4=44∕3s,, or ä = 4/3Î = i\*75. , The actual value of *ħ* for the moon and earth is about 4; hence, if the moon-earth system were started with less than ∕3 of its actual moment of momen­tum, it would not be possible for the two bodies to move so that the earth should always show the same face to the moon. Again, if we travel along the line of , momentum, there must be some point for which *yxi* is a maximum, and since *yx\*≈n∣ω* there,must be some point for which the number of planetary rotations is greatest during one revolution of the satellite; or, shortly, there must be some configura­tion for which there is a maximum number of days in the month. Now yx3 is equal to x3 (Ä—x), and this is a maximum when x = |Ä and the maximum number of days in the month is (J⅛)3(A-1⅛) or 33⅛4∕44 ; if *h* is equal to 4, as is nearly the case for the earth and moon, this becomes 27. Hence it follows that we now have very nearly the maximum number of days in the month. A more accurate investigation in a paper on the “ Precession of a Viscous Spheroid ” in *Phil. Trans.* (1879), pt. i., showed that, taking account of solar tidal friction and of the obliquity to the ecliptic, the maximum number of days is,about 291 and that we have already passed through the phase of maximum.

We will now consider the physical meaning of the figure. It is assumed that the resultant moment of momentum of the whole system corresponds to a positive rotation. Now imagine two points with the same abscissa, one on the momentum line and the other on the energy curve, and suppose the one on the energy,curve,to,guide that on the momentum line. Since we are supposing frictional tides to be raised on the planet, the energy must degrade, and however the two points are set initially the point on the energy curve must always slide down a slope, carrying with it the other point. Looking at the figure, we see that there are four slopes in the energy curve, two running down to the planet and two down to the minimum. There are therefore four ways in which the system may degrade, according to the way it was started; but we shall only consider one, that corresponding to the portion ABZ>α of the figure. For the part of the line of momentum AB the month is longer than the day, and this is the case with, all known satellites except the nearer one of Mars. Now, if a satellite be placed in the condition A—that is to say, moving rapidly round a planet which always shows the same face to the satellite— the condition is clearly dynamically unstable, for the least distur­bance will determine,whether the system shall degrade down the slopes *ac* or *ab—*that is to say, whether it falls into or recedes from the planet. If the equilibrium breaks down by the satellite receding, the recession will go on until the system, has reached the state corresponding to B. It is clear that, if the intersection of the edge of the shaded strip with the line of momentum be identical with the point A, which indicates that the satellite is just touching the planet, then the two bodies are in effect parts of a single body in an unstable configuration. If, therefore, the moon was originally part of the earth, we should expect to find this identity. Now in fig. 9, drawn to scale to represent the earth and moon, there is so close an approach between the edge of the shaded band,and the intersec­tion of the line of momentum and curve of rigidity that it would be scarcely possible to distinguish them. Hence, there, seems a probability that the two bodies once formed parts, of a single one, which broke up in consequence of some kind of instability. This view is confirmed by the more detailed consideration, of the case in the paper on the u Precession of a Viscous Spheroid,” already referred to, and subsequent papers, inthePÄîZ. *Trans.@@1*

§ 36. *Effects of Tidal Friction on the Elements of the Moon's Orbit and on the Earth's Rotation.—*It would be impossible within the limits of the present article to discuss completely the effects of tidal friction; we therefore confine ourselves to certain general considerations which throw light on the nature of those effects. We have in the preceding section supposed that the planet’s axis is perpendicular to the orbit of the satellite, and that the latter is circular; we shall now suppose the orbit to be oblique to the equator and eccentric. For the sake of brevity the planet will be, called the earth, and the satellite the moon. The complete investigation was carried out on the hypothesis that the planet, was a viscous spheroid, because this was the only theory of lrictionally resisted tides which had been worked out. Although the results would be practically the same for any system of fπctionally resisted tides, we shall speak below of the planet or earth as a viscous body.@@2

We shall show that if the tidal retardation be small the obliquity of the ecliptic increases, the earth’s rotation is retarded, and the moon’s, distance and peri­odic time are increased. Fig. 9 represents the earth as seen from above, the south pole, so that S is the pole and the outer circle the equator. The earth’s rotation is in the direc­tion of the curved arrow, at ,S. The half of the inner circle which is drawn with a full line is a semi-small-circle of south latitude, and the, dotted semicircle is a semi-small-circle in the same north latitude. Generally dotted lines indicate parts of the figure which are below the plane of the paper. If the moon were cut in two and one half retained at the place of the moon and the other half transported to a point diametrically opposite to the first half with reference to the earth, there would be no material change in the tide-generating forces. It is easy to verify this statement by reference to § 11. These two halves may be,described as moon and anti-moon, and such a substitution will facilitate the,explanation. Let M and M' be the projections of the,moon and anti-moon on to the terrestrial sphere. If the fluid in which the tides are raised were perfectly frictionless,@@3 or if the earth were a perfect fluid or perfectly elastic, the apices of the tidal spheroid would be at M and M'. If, however, there is internal friction, due to any sort of viscosity, the tides will lag, and we may suppose the tidal apices to be at T and T'. Now suppose the tidal protuberances to be replaced by,two equal heavy particles at T and T', which are, instantaneously rigidly con­nected with the earth. Then the attraction of the moon on T is greater

@@@1 For further consideration of this subject see a series of papers by G. H. Darwin in *Proceed,* and *Trans,* of the Royal Society from 1878 to 1881, and aρρ. G. (ò) t. pt. ii. vol. i. of Thomson and Tait’s 2VαL *Phil.* (1883); or *Scientific Papers,* vol. ii.

@@@2 These explanations, together with other remarks, are to be found in the abstracts of G. H. Darwin’s memoirs in *Proc. Roy. Soc.t* 1878 to 1881.

@@@3 We here suppose the tides not to be inverted. If they are inverted the conclusion is precisely the same.