than on T', and that of the anti-moon on T' is greater than on T. The resultant of these forces is clearly a pair of forces acting on the earth in the direction TM, T'M'. These forces cause a couple about the axis in the equator, which lies in the same meridian as the moon and anti-moon. The direction of the couple is shown by the curved arrows at L,L'. If the effects of this couple be compounded with the existing rotation of the earth according to the principle of the gyroscope, the south pole S will tend to approach M and the north pole to approach M'. Hence, supposing the moon to move in the ecliptic, the inclination of the earth’s axis to the ecliptic dimin­ishes, or the obliquity increases. Next the forces TM, T'M' clearly produce, as in the simpler case considered in § 9, a couple about the earth’s polar axis, which tends to retard the diurnal rotation.

This general explanation remains a fair representation of the state of the case so long as the different harmonic constituents of the aggregate tide-wave do not suffer very different amounts of retardation; and this is the case so long as the viscosity is not great. The rigorous result for a viscous planet shows that in general the obliquity will increase, and it appears that, with small viscosity of the planet, if the period of the satellite be longer than two periods of rotation of the planet, the obliquity increases, and vice versâ. Hence, zero obliquity is only dynamically stable when the period of the satellite is less than two periods of the planet’s rotation.

It is possible, by similar considerations, to obtain some insight into the effect which tidal friction must have on the plane of the lunar orbit, but as the subject is somewhat complex we shall not proceed to a detailed examination of the question. It must suffice to say that in general the inclin­ation of the lunar orbit must diminish. Now let us con­sider a satellite revolving about a planet in an elliptic orbit, with a periodic time which is long compared with the period of rotation of the planet; and suppose that frictional tides are raised in the planet. The major axis of the tidal spheroid always points in advance of the satellite, and exercises on it a force which tends to accelerate its linear velocity. When the satellite is in perigee the tides are higher, and this disturbing force is greater than when the satellite is in apogee. The disturbing force may therefore be repre­sented as a constant force, always tending to accelerate the motion of the satellite, to which is added a periodic force accelerating in perigee and retarding in apogee. The constant force causes a secular increase of the satellite’s mean distance and a retardation of its. mean motion. The accelerating force in perigee causes the satellite to swing out farther than it would otherwise have done, so that when it comes round to apogee it is more remote from the planet. The retarding force in apogee acts exactly inversely, and diminishes the perigean distance. Thus, the apogean distance in­creases and the perigean distance diminishes, or in other words, the eccentricity of the orbit increases. Now consider another case, and suppose the satellite’s periodic time to be identical with that of the planet’s rotation. Then, when the satellite is in perigee, its angular motion is faster than that of the planet’s rotation, and when in apogee it is slower; hence at apogee the tides lag, and at perigee they are accelerated.. Now the lagging apogean tides give rise to an accelerating force on the satellite, and increase the peri­gean distance, whilst the accelerated perigean tides give rise to a. retarding force, and decrease the apogean dis­tance. Hence in this case the eccentricity of the orbit will diminish. It follows from these two results that there must be some inter­mediate periodic time of the satellite for which the eccentricity does not tend to vary.

But the preceding general explanation is in reality somewhat less satisfactory than it seems, because it does not make clear the exist­ence of certain antagonistic influences, to which, however, we shall not refer. The full investigation for a viscous planet shows that in general the eccentricity of the orbit will increase. When the viscos­ity is small the law of variation of eccentricity is very simple : if eleven periods of the satellite occupy a longer time than eighteen rotations of the planet, the eccentricity increases, and vice versâ. Hence in the case of small viscosity a circular orbit is only dynamically stable if the eleven periods are shorter than the eighteen rotations.

VIII.—Cosmogonic Speculations Founded on Tidal Friction

§ 37. *History of the Earth and Moon.—*We shall not attempt to discuss the mathematical methods by which the complete history of a planet, attended by one or more satellites, is to be traced. The laws indicated in the preceding sections show that there is such a problem, and that it may be solved, and we refer to G. H. Darwin’s papers for details *(Phil. Trans.*, 1879-1881). It may be interesting, however, to give the various results of the investigation in the form of a sketch of the possible evolution of the earth and moon, fol­lowed by remarks on the other planetary systems and on the solar system as a whole.

We begin with a planet not very much more than 8000 m. in diameter, and probably partly solid, partly fluid, and partly gaseous. It is rotating about an axis inclined at about 11° or 12° to the normal to the ecliptic, with a period of from two to four hours, and is revolving about the sun with a period not much shorter than our present year. The rapidity of the planet’s rotation causes so great a compression of its figure that it cannot continue to exist in an ellipsoidal form with stability; or else it is so nearly unstable that complete instability is induced by the solar tides. The planet then separates into two masses, the larger being the earth and the smaller the moon. It is not attempted to define the mode of separation, or to say whether the moon was initially a chain of meteorites. At any rate it must be assumed that the smaller mass became more or less conglomerated and finally fused into a spheroid, perhaps in consequence of impacts between its constituent meteor­ites, which were once part of the. primeval planet. Up to this point the history is largely speculative, for the investigation of the conditions of instability in such a case surpasses the powers of the mathematician. We have now the earth and moon nearly in con­tact with one another, and rotating nearly as though they were parts of one rigid body. This is the system which was the subject of dynamical investigation. As the two masses are not rigid, the attraction of each distorts the other; and, if they do not move rigorously with the same periodic time, each raises a tide in the other. Also the sun raises tides in both. In consequence of the frictional resist­ance to these tidal motions, such a system is dynamically unstable. If the moon had moved orbitally a little faster than the earth rotated, she must have fallen back into the earth; thus the exist­ence of the moon compels us to believe that the equilibrium broke down by the moon revolving orbitally a little slower than the earth rotates. In consequence of the tidal friction the periodic times both of the moon (or the month) and of the earth's rotation (or the day) increase; but the month increases in length at a much' greater rate than the day. At some early stage in the history of the system the moon was conglomerated into a spheroidal form, and acquired a rotation about an axis nearly parallel to that of the earth.

The axial rotation of the moon is retarded by the attraction of the earth on the tides raised in the moon, and this retardation takes place at a far greater rate than the similar retardation of the earth’s rotation. As soon as the moon rotates round her axis with twice the angular velocity with which she revolves in her orbit, the position of her axis of rotation (parallel with the earth’s axis) becomes dynamically unstable. The obli­quity of the lunar equator to the plane of the orbit increases, attains a maximum, and then diminishes. Meanwhile the lunar axial rotation is. being reduced towards identity with the orbital motion. Finally, her equator is nearly coincident with the plane of the orbit, and the attraction of the earth on a tide, which degene­rates into a permanent ellipticity of the lunar equator, causes her always to show the same face to the earth.

All this must have taken place early in the history of the earth, to which we now return. At first the month is identical with the day, and. as. both these increase in length the lunar orbit will retain its circular form until the month is equal to 1 7/11 @@1 days. From that time the orbit begins to be eccen­tric, and the eccentricity increases thereafter up to its present magnitude. The plane of the lunar orbit is at first practically identical with the earth’s equator, but as the moon recedes from the earth the sun’s attraction begins to make itself felt. We shall not attempt to trace the complex changes by which the plane of the lunar orbit is. affected. It must suffice to say that the present small inclination of the lunar orbit to the ecliptic accords with the theory.

As soon as the earth rotates with twice the angular velocity with which the moon revolves in her orbit, a new instability sets in. The month is then about twelve of our present hours, and the day about six. such hours in length. The inclination of the equator to the ecliptic now begins to increase and continues to do so until finally it reached its present value of 23½°. All these changes continue and.no new phase now supervenes, and at length we have the system in its present configuration. The minimum time in which the changes from first to last can have taken place is 54,000,000 years.

There are other collateral results which must arise from a supposed primitive viscosity or plasticity of the earth’s mass. For during this course of evolution the earth’s mass must have suffered a screwing motion, so that the polar regions have travelled a little from west to east relatively to the equator. This affords a possible explanation of the north and south trend of our great continents. The whole of this argument reposes on the imperfect rigidity of solids and on the internal friction of semi-solids and fluids; these are *verae causae.* Thus changes of the kind here discussed must be going on, and must have gone on in the past. And for this history of the earth and moon to be true throughout, it is only necessary to postulate a sufficient lapse of time, and that there is not enough matter diffused through space materially to resist the motions of the moon and earth in perhaps 200,000,000 years. It seems hardly too much to say that, granting these two postulates, and the existence of a

@@@1 See criticism, by Nolan, *Genesis of Moon* (Melbourne, 1885); also *Nature* (Feb. 18, 1886).