clearer perception of the importance of knowing the accurate time of phenomena, and in the year 829 we find it stated that at the commencement of the solar eclipse on the 30th of November the altitude of the sun was 70 and at the end 240, as observed at Bagdad by Ahmed ibn Abdallah, called Habash.@@1 This seems to be the earliest determination of time by an altitude; and this method then came into general use among the Arabians, who, on observing lunar eclipses, never failed to measure the altitude of some bright star at the beginning and end of the eclipse. In Europe this method was adopted by Purbach and Regiomontanus apparently for the first time in 1457 Bernhard Walther, a pupil of the latter, seems to have been the first to use for scientific purposes clocks driven by weights: he states that on the 16th of January 1484 he observed the rising of the planet Mercury, and immediately attached the weight to a clock having an hour- wheel with fifty-six teeth; at sunrise one hour and thirty-five teeth had passed, so that the interval was an hour and thirty­seven minutes. For nearly two hundred years, until the applica­tion of the pendulum to clocks became general, astronomers could place little or no reliance on their clocks, and conse­quently it was always necessary to fix the moment of an observa­tion by a simultaneous time determination. For this purpose Tycho Brahe employed altitudes observed with quadrants; but he remarks that if the star is taken too near the meridian the altitude varies too slowly, and if too near the horizon the refrac­tion (which at that time was very imperfectly known) introduces an element of uncertainty. He sometimes used azimuths, or with the large “ armillary spheres ” which played so important a part among his instruments, he measured hour-angles or distances from the meridian along the equator.@@2 Transits of stars across the meridian were also observed with the meridian quadrant, an instrument which is alluded to by Ptolemy and was certainly in use at the Marãgha (Persia) observatory in the 13th century, but of which Tycho was the first to make extensive use. But he chiefly employed it for determining star-places, having obtained the clock error by the methods already described.

In addition to these methods, that of “ equal altitudes ” was much in use during the 17th century. That equal distances east and west of the meridian correspond to equal altitudes had of course been known as long as sundials had been used; but, now that quadrants, cross-staves and parallactic rules were commonly employed for measuring altitudes more accurately, the idea naturally suggested itself to determine the time of a star's or the sun’s meridian passage by noting the moments when it reached any particular altitude on both sides of the meridian. But Tycho’s plan of an instrument fixed in the meridian was not forgotten, and from the end of the 17th century, when Römer invented the transit instrument, the observation of transits across the meridian became the principal means of determining time at fixed observatories, while the observation of altitudes, first by portable quadrants, afterwards by reflecting sextants, and during the 19th century by portable alt-azimuths or theodolites, has been used on journeys. Since about 1830 the small transit instrument, with what is known as a “ broken telescope,” has also been much employed on scientific expedi­tions; but great caution is necessary in using it, as the difficulties of getting a perfectly rigid mounting for the prism or mirror which reflects the rays from the object glass through the axis to the eyepiece appear to be very great, for strange discrepan­cies in the results have often been noticed. The gradual develop­ment of astronomical instruments has been accompanied by a corresponding development in timekeepers. From being very untrustworthy, astronomical clocks are now made to great perfection by the application of the pendulum and by its com­pensation, while the invention of chronometers has placed a portable and equally trustworthy timekeeper in the hands of travellers.

We shall now give a sketch of the principal methods of determining: time.

In the spherical triangle *ZPS* between the zenith, the pole and a star the side *ZP=yoa-φ (φ* being the latitude), PS=90o-- S (.δ being the declination), and *ZS* 0rz=90o minus the observed altitude. The angle *ZPS — t* is the star’s hour-angle or, in time, the interval between the moment of observation and the meridian passage of the star. We have then

cos í — sin <⅛ sin δ cos *t = ———,— —; ,*

cos *φ* cos *i ’*

which formula can be made more convenient for the use of logarithms by putting *z+Φ+6 = 2S,* which gives

tan2H=~~≠~~ ~~(5~~~~~~~~~φ)~~ ~~T.~~ ~~(~~~~V~~~~}~~~~·~~

• cos ó cos (ó — z)

According as the star was observed west or east of the meridian, *I* will be positive or negative. If α be the right ascension of the star, the sidereal time —i-!-α, *a* as well as δ being taken from an ephemeris. If the sun had been observed the hour-angle *t* would be the apparent solar time. The latitude observed must be cor­rected for refraction, and in the case of the sun also for parallax, while the sun’s semi-diameter must be added or subtracted accord­ing as the lower or upper limb was observed. The declination of the sun being variable, and being given in the ephemerides for noon of each day, allowance must be made for this by interpolating with an approximate value of the time. As the altitude changes very slowly near the meridian, this method is most advantageous if the star be taken near the prime vertical, while it is also easy to see that the greater the latitude the more uncertain the result. If a number of altitudes of the same object are observed, it is not necessary to deduce the clock error separately from each observa­tion, but a correction may be applied to the mean of the zenith distances. Supposing *n* observations to be taken at the moments *Tι, Pi, Tt,.* the mean of all being To, and calling the z corresponding to this *Z,* we have

≡ι=-Z + ⅛T1-T0)+i⅛(T1-T0)2ι *⅞ = Z +* - Γo) + 21⅞(Γ1J- To)2;

and so on, í being the hour-angle answering to To. As Σ(T- T0)=o, these equations give

<z \_ ⅞ ^b⅞ + ¾ + ■.. \_ I *<PZ* (T1 — Tp)2 -p (T2 — To)2 -j- .. .

*n* 2 *iti n ’*

\_ Zι +zq + z> + ... *<PZ* ∑2 sin2\* (T — Tc)

*n - dti n*

But, if in the above-mentioned triangle we designate the angles at *Z* and 5 by 18oo- *A* and *p,* we have

sin z sin *A* =cos δ sin í;

sin z cos *A =* —cos *φ* sin δ + sin *φ* cos δ cos í; and by differentiation

*<PZ \_* cos *φ* cos δ cos *A* cos *p dl2 ~* sin Z ,

in which *A* and *p* are determined by

. . sin *t .* 1 . sin *t*

sin *A = ■* 7 cos δ and sin *p = ' i —*cos *φ.*

With this corrected mean of the observed zenith distances the hour- angle and time are determined, and by comparison with To the error of the timekeeper.

The method of equal altitudes gives very simply the clock error equal to the right ascension minus half the sum of the clock times corresponding to the observed equal altitudes on both sides of the meridian. When the sun is observed, a correction has to be applied for the change of declination in the interval between the observations. Calling this interval 2/, the correction to the apparent noon given by the observations *x,* the change of declination in half the interval ∆δ, and the observed altitude ⅛, we have

sin ⅛ = sin *φ* sin (δ-∆δ)+cos *φ* cos (δ-∆δ) cos (∕-∣-x) and sin ⅛ = sin *φ* sin (δ⅛∆δj+cos.≠ cos (δ+∆δ) cos (Z—x), whence, as cos *x* may be put = 1, sin *x = x,* and tan ∆δ =∆δ,

/tan *φ* tan δ∖ ..

*x ~ ∖sint* tan *t ∕iiδ,*

which, divided by 15, gives the required correction in seconds of time. Similarly an afternoon observation may be combined with an observation" made the following morning to find the time of apparent midnight.

The observation of the time, when a star has a certain azimuth may also be used for determining the clock error, as the hour-angle can be found from the declination, the latitude and the azimuth. As the azimuth changes most rapidly at the meridian, the observa­tion is most advantageous there, besides which it is neither necessary to know the latitude nor the declination accurately. The observed time of transit over the meridian must be corrected for the deviations of the instrument in azimuth, level and collimation. This corrected time of transit, expressed in sidereal time, should then be equal to the right ascension of the object observed, and the difference is the clock error. In observatories the determination of a clock’s error (a necessary operation during a night’s work with a transit

@@@1 Caussin, *Le Livre de la grande table Hakémite,* p. 100 (Paris, 1804).

@@@2 bee his *Epistolae astronomicae,* p. 73.