has increased by ageing. If so, it may mean a very considerable increase in the cost of magnetizing power. Consider the case of a 30-kilowatt transformer connected to the mains all the year round; the normal core loss of such a transformer should be about 300 watts, and therefore, since there are 8760 hours in the year, the total annual energy dissipated in the core should be 2628 kilowatt hours. Reckoning the value of this electric energy at only one penny per unit, the core loss costs £10, 19s. per annum. If the core loss becomes doubled, it means an additional annual expenditure of nearly £11*.* Since the cost of such a transformer would not exceed £100, it follows that it would be economical to replace it by a new one rather than continue to work it at its enhanced core loss.

In Great Britain the sheet steel or iron alloy used for the trans­former cores is usually furnished to specifications which state the maximum hysteresis loss to be allowed in it in watts per lb (avoirdupois) at a frequency of 50, and at a maximum flux-density during the cycle of 4000 C.G.S. units. When plates having a thickness *t* mils are made up into a transformer core, the total energy loss in the core due to hysteresis and eddy current loss when worked at a frequency *n* and a maximum flux-density during the cycle B is given by the empirical formulae

T = ∙0032*n*B1·5510-7+(*tn*B)210-l6, T1=0∙88*n*B11·5510-9+1∙4(*t*1*n*B1)21010, where T stands for the loss per cubic centimetre, and T1 for the same in watts per pound of iron core, B for the maximum flux- density in lines per square centimetre, and B1 for the same in lines per square inch, *t* for the thickness of the plates in thousandths of an inch (mils), and *t*1 for the same in inches. The hysteresis loss varies as some power near to 1∙6 of the maximum flux-density during the cycle as shown by Steinmetz (see Electromagnetism). Since the hysteresis lose vanes as the 1∙6th power of the maximum flux density during the cycle (B max.), the advantages of a low flux-density are evident. An excessively low flux-density increases, however, the cost of the core and the copper by increasing the size of the transformer. If the form factor (*f*) of the primary voltage curve is known, then the maximum value of the flux-density in the core can always be calculated from the formula B=E1/4*fn*SN1, where E is the R.M.S. value of the primary voltage, N1 the primary turns, S the section of the core, and *n* the frequency.

The study of the processes taking place in the core and circuits of a transformer have been greatly facilitated in recent years by the improvements made in methods of observing and recording the variation of periodic currents and electromotive forces. The original method, due to Joubert, was greatly improved and employed by Ryan, Bell, Duncan and Hutchinson, Fleming. Hopkinson and Rosa, Callendar and Lyle; but the most important improvement was the introduction and invention of the oscillograph by Blondel, subsequently improved by Duddell, and also of the ondograph of Hospitalier (see Oscillograph). This instrument enables us, as it were, to look inside a transformer, for which it, in fact, performs the same function that a steam engine indicator does for the steam cylinder.@@1 Delineating in this way the curves of primary and secondary current and primary and secondary electromotive forces, we get the following result: Whatever may be the form of the curve of primary terminal potential difference, or primary voltage, that of the secondary voltage or terminal potential difference is an almost exact copy, but displaced 180° in phase. Hence the alternating current trans­former reproduces on its second­ary terminals all the variations of potential on the primary, but changed in scale. The curve of primary current when the transformer is an open secondary circuit is different in form and phase, lagging behind the primary voltage curve (fig. 6); but if the transformer is loaded up on its

secondary side, then the primary current curve comes more into step with the primary voltage curve. The secondary current curve, if the secondary load is non-inductive, is in step with the secondary voltage curve (fig. 7). These transformer diagrams yield much information as to the nature of the operations proceeding in the transformer.

The form of the curve of primary current at no secondary load is a consequence of the hysteresis of the iron, combined with the fact that the form of the core flux-density curves of the transformer is always not far removed from a simple sine curve. If *e*1 is at any moment the electromotive force, *i*1 the current on the primary circuit, and *b*1 is the flux-density in the core, then we have the fundamental relation *e*1 = R1*i*1+SN1 *db*1*∣dt,* where R1 is the resistance of the primary, and N1 the number of turns, and S is the cross-section of the core. In all modern closed circuit trans- formers the quantity R1*i*1 is very small compared with the quantity SN*db∣dt* except at one instant during the phase, and in taking the integral of the above equation, viz. in finding the value of *fe*1*dt,* the integral of the first term on the right-hand side may be neglected in comparison with the second. Hence we have approxi- mately *b*1= (SN1)-1∫*e*1*dt*. In other words, the value of the flux- density in the core is obtained by integrating the area of the primary voltage curve. In so doing the integration must be started from the time point through which passes the ordinate bisecting the area of the primary voltage curve. When any curve is formed such that its ordinate *y* is the integral of the area of another curve, viz. *y=∫y*1*dx,* the first curve is always smoother and more regular in form than the second. Hence the process above described when applied to a complex periodic curve, which can by Fourier’s theorem be resolved into a series of simple periodic curves, results in a relative reduction of the magnitude of the higher harmonics compared with the fundamental term, and hence a wiping out of the minor irregularities of the curve. In actual practice the curve of electromotive force of alternators can be quite sufficiently reproduced by employing three terms of the expansion, viz. the first three odd harmonics, and the resulting flux-density curve is always very nearly a simple sine curve.

We have then the following rules for predetermining the form of the current curve of the transformer at no load, assuming that the hysteresis curve of the iron is given, set out in terms of flux-density and ampere-turns per centimetre, and also the form of the curve of primary electromotive force. Let the time base line be divided up into equal small elements. Through any selected point draw a line perpendicular to the base line. Bisect the area enclosed by the curve representing the half wave of primary electromotive force and the base line by another perpendicular. Integrate the area enclosed between the electromotive force curve and these two perpendicular lines and the base. Lastly, set up a length on the last perpendicular equal to the value of this area divided by the product of the cross-section of the core and the number of primary turns. The resulting value will be the core flux-density *b* at the phase instant corresponding. Look out on the hysteresis loop the same flux-density value, and corresponding to it will be found two values of the magnetizing force in ampere-turns per centimetre, one the\* value for increasing flux-density and one for decreasing. An inspection of the position of the point of time selected on the time line will at once show which of these to select. Divide that value of the ampere-turns per centimetre by the product of the values of the primary turns and the mean length of the magnetic circuit of the core of the transformer, and the result gives the value of the primary current of the transformer. This can be set up to scale on the perpendicular through the time instant selected. Hence, given the form of the primary electromotive force curve and that of the hysteresis loop of the iron, we can draw the curves representing the changes of flux-density in the core and that of the corresponding primary current, and thus predict the root- mean-square value of the magnetizing current of the transformer. It is therefore possible, when given the primary electromotive force curve and the hysteresis curve of the iron, to predetermine the curves depicting all the other variables of the transformer, provided that the magnetic leakage is negligible.

The elementary theory of the closed iron circuit transformer may be stated as follows: Let N1, N2 be the turns on the primary and secondary circuits, R1 and R2 the resistances, S the section of the core, and *b*1 and the co-instantaneous values of the flux-density just, inside, the primary and secondary windings. Then, if *i*1 and *i*2 and *e*1 and *e*2 are the primary

@@@1 For a useful list of references to published papers on alternating current curve tracing, see a paper by W. D. B. Duddell, read before the British Association, Toronto, 1897; also *Electrician* (1897), xxxix. 636; also *Handbook for the Electrical Laboratory and Testing Room* (J. A. Fleming), i. 407.