of gravity in each ſection is to be found. Let AB be the line of floatation of the ſhip when in an upright ſtate, and *a b* the water line when inclined. Then, becauſe the weight of the ſhip remains the ſame, the quantity of water displaced will alſo be the ſame in both caſes, and therefore AEB — aEb, each ſuſtaining the ſame part of the whole weight of the ſhip. From each of theſe take the part AE h, which is common to both, and the remainders AO b, BO *b* will be equal ; and which, becauſe the inclination is ſuppoſed very small, may be conſidered as rectilineal triangles, and the point O the middle of AB.

Now, let H, I, K, be the centres of gravity of the ſpaces A O a, AE *b,* and BO b*,* reſpectively. From theſe points draw the lines H *h,* I i, and K *k,* perpen­dicular to AB, and let IL be drawn perpendicular to EO. Now to aſcertain the diſtance γ *q* of the centre of gravity *y* of the part *a* E *b* ſrom the line AB, the momentum of *a* E *b* with respect to this line muſt be put equal to the difference of the momentums of the parts AE *b,* AO *a,* which are upon different ſides of AB @@\*. Hence a E *b* X γ *q,* or AEB × γ q = AE *b* X I *i—*AO *a* × H *h.* But ſince *g* is the common centre of gravity of the two parts AEb, BO *b,* we have there­fore AEB × gO = AEi×I ;+BO *b× Kk.* Hence by expunging the term AE *b* × I *i* from each of theſe equations, and comparing them, we obtain AEB X γ q = AEB× g O — BO *b* X K *k —* AO *a ×* H h.

Now, ſince the triangles AO *a,* BO *b,* are ſuppoſed infinitely ſmall, their momentums or products, by the infinitely little lines H *h,* K *k,* will alſo be infinitely ſmall with reſpect to AEB X *g* O; which therefore be­ing rejected, the former equation becomes AEB × γ *q* = AEBx g O, and hence γq = gO. Whence the centres of gravity γ*,* g, being at equal diſtances below AB, the infinitely little line *y g* is therefore perpendi­cular to EO. For the ſame reaſon g γ, fig. 52. may be conſidered as an arch of a circle whole centre is M.

To determine the value of *g y,* the momentum of ***a*** E *b* with reſpect to EO muſt be taken, for the ſame reaſon as before, and put equal to the momentums of the two parts AO *a,* AE *b* ; and we ſhall then have *a* E *b × gγ,* or AEB X *g γ* = AEB X IL + AO *a* χ O A But ſince *g* is the oommon centre of gravity of the two ſpaces AE *b,* BO *b,* we ſhall have AE *b* X IL — BOb X *Ok =* O, or AEb X IL = BO b × Ok. Hence AEB × g *γ* = BOb × Ok +AOa × Oh

— 2 BOb X Ok ; becauſe the two triangles AO *a,* BO *b* are equal, and that the diſtances O*k,* O h, are alſo evidently equal.

Let x be the thickneſs of the ſection repreſented by ABC. Then the momentum of this ſection will be 2 BO *b* X x X O*k,* which equation will alſo ſerve for each particular ſection.

Now let s' repreſent the ſum of the momentums of all the ſectior.s. Hence *ſ,* AEB × x X *g γ = s,* 2. BO *b* X x X O *k.* Now the firſt member being the ſum of the momentums of each ſection, in proportion to a plane paſſing through the keel, ought therefore to be equal to the ſum of all the ſections, or to the volume of the immerſed part of the bottom multiplied by the diſtance *g y.* Hence V repreſenting the volume, we ſhall have V X *g γ = ſ, 2* BO *b* X *x* O*k.*

In order to determine the value of the ſecond member of this equation, it may be remarked, that when the ſhip is inclined, the original plane of floatation CBPQ, (fig. 54.) becomes C *b ρ* Q. Now the triangles NIn, BO *b,* being the ſame as thoſe in figures 52∙ and 53 ; and as each of theſe triangles have one angle equal, they may, upon account of their infinite ſmallneſs, be conſi­dered as ſimilar ; and hence BOb:NI *n* : : OB

QB

: IN ; whence BO b =, X N In. Moreover, we

IN

have (fig. 53.) O*k —* 2/3 OB, for the points K and *k* may be conſidered as equidiſtant from the point O :

whence BOb x Ok = ×NI n.

IN\*

Hence V ×g*γ = s,* 3/4OB X x × NIn. From this IN∣2

equation the value of *g y* is obtained.

To find the altitude *g* M (fig. 55.) of the meta­center above the centre of gravity of the immerſed part of the bottom, let the arc NS be deſcribed from the centre I with the radius IN ; then NI *n = IN X NS. Now*

ſince the two ſtraight lines *γ* M, *g* M are perpendicular to *an* and AN respectively, the angles M and NI *n* are therefore equal : and the infinitely little portion g*γ,* which is perpendicular to *g* M, may be conſidered as an arch deſcribed from the centre M. Hence the two ſectors NTS, *g* M *y* are ſimilar ; and therefore *g* M: g*γ:* IN : NS. Hence NS = IN x g*γ* ; and conſequently

gM

NI *n* Now this being ſubſtituted in the

2 g M

former equation, and reduced, we have V X g*γ = s*

*2/3OB X x X gγ*

-. But ſince *g* M and *g y* are the

*g* M 0 .

ſame, whatcver ſection may be under conſideration, the equation may therefore be expreſſed thus, V ×g =

2/3 g*γ* . s , OBl3× V. Hence *g M* = 2/3s*γOB x*

*g* Μ V

Let *y =* OB, and the equation becomes g M =

2 *r* 3

2/3s, y3xWhence to have the altitude of the metacenter above the centre of gravity of the immerſed part of the bottom, the length of the ſection at the water­line muſt be divided by lines perpendicular to the middle line of this ſection into a great number of equal parts, ſo that the portion of the curve contained between any two adjacent perpendiculars may be conſidered as a ſtraight line. Then the ſum of the cubes of the half perpendiculars or ordinates is to be multiplied by the diſtance between the perpendiculars, and two thirds of the product is to be divided by the volume of the im­merſed part of the bottom of the ſhip.

It is hence evident, that while the ſector at the wa­ter line is the ſame, and the volume of the immerſed part of the bottom remains also the ſame, the altitude of the metacenter will remain the ſame, whatever may be the figure of the bottom.

Chap. IV. *Of the Centre of Gravity of the immerſed Part oſ the Bottom oſ a ship.*

The centre of gravity@@\* of a ſhip, ſuppoſed homo­geneous, and in an upright poſition in the water, is in a

@@@[m]\* Bezout's Mechanique, art. 263.

@@@[m]\* See Mechanics.