Call their diſtances from the axis of fracture *o* and *g.* Then AI or i = og/d, and the momentum of coheſion is f*s × og/d*, where s is the area of fracture.

This index is eaſily determined in all the cases which generally occur in practice. In a rectangular beam AI is 1/3d of AD ; in a cylinder (circular or elliptic) AI is 5/10ths of AD, &c. &c.

In this hypotheſis, that the coheſion actually exerted by each fibre is as its extension, and that the extenſions of the fibres are as their diſtances from A (fig. 5. n⁰ 1.), it is plain that the forces exerted by the fibres D, E, &c. will be repreſented by the ordinates D d*,* E *e,* &c. to a ſtraight line A *d.* And we learn from the principles of Rotation that the centre of percuſſion O is in the ordinate which paſſes through the centre of gravity of the triangle A D *d,* or (if we conſider the whole ſection having breadth as well as depth) through the centre of gravity of the ſolid bounded by the planes DA, *d* A ; and we found that this point O was the centre of effort of the coheſions *actually* exerted in the inſtant of fracture, and that I was the centre of an *equal* momentum, which would be produced if all the fibres were accumulated there and exerted their f*ull* coheſion.

This conſideration enables us to determine, with equal facility and neatneſs, the ſtrength of a beam in any hypothesis of forces. The above hypotheſis was introduced with a cautious limitation to moderate ſtrains, which produced no permanent change of form, or no ſett as the artiſts call it : and this ſuffices for all purpoſes of practice, seeing that it would be imprudent to expoſe materials to more violent strains. But when we compare this theory with experi­ments in which the pieces are really broken, conſiderable deviations may be expected, becauſe it is very probable that in the vicinity of rupture the forces are no longer pro­portional to the extenſions.

That no doubt may remain as to the juſtneſs and completeneſs of the theory, we muſt ſhow how the relative ſtrength may be determined in any other hypotheſis. There­fore ſuppoſe that it has been eſtabliſhed by experiment on any kind of ſolid matter, that the forces actually exerted in the inſtant of fracture by the fibres at D, E, &c. are as the ordinates D *d',* E *e, &c.* of any curve line A *e' d'.* We are suppoſed to know the form of this curve, and that of the solid which is bounded by the vertical plane through AD, and by the ſurface which paſſes through this curve A *e' d'* perpendicularly to the length of the beam. We know the place of the centre of gravity of this curve ſurface or ſolid, and can draw a line through it parallel to AB, and cutting the ſurface of fracture in ſome point O. This point is alſo the centre of effort of all the coheſions actually exerted ; and the product of AO and of the ſolid which expresses the actual coheſions will give the momentum of coheſion equivalent to the former *fs* og/d. Or we may find an index A I, by making AI a fourth proportional to the full cohe­ſion of the ſurface of fracture, to the accumulated actual cohe­ſions, and to AO ; and then f*s × i* (=AI) will be the momentum of coheſion ; and we ſhall ſtill have I for the point in which all the fibres may be fuppoſed to exert their full coheſion f*,* and to produce a momentum of coheſion equal to the real momentum of the coheſions actually exerted, and the relative ſtrength of the beam will ſtill be p = fsi/l or fsgo/dl.

Thus, if the forces be as the ſquares of the extensions (ſtill ſuppoſed to be as the diſtances from A), the curve A e' d' will be a common parabola, having AB for its axis and AD for the tangent at its vertex. The area AD *d'* will be 1/3d AD × D*d;* and in the caſe of a rectan­gular beam, AO will be 3/4ths AD, and AI will be 1/4th of AD.

We may obſerve here in general, that if the forces actually exerted in the inſtant of fracture be as any power q AD of the diſtance from A, the index AI will be = AD/q+2 for a rectangular beam, and the momentum of coheſion will always be (ca*eteris paribus)* as the breadth and as the ſquare of the depth ; nay, this will be the caſe whenever the action of the fibres D and E is expreſſed by any *ſimilar functions* of *d* and *x.* This is evident to every reader acquainted with the fluxionary calculus.

As far as we can judge from experience, no ſimple algebraic power of the diſtance will expreſs the actual coheſions of the fibres. No curve which has either AD or AB for its tangent will suit. The obſervations which we made in the beginning ſhow, that although the curve of fig. 2. muſt be ſenſibly ſtraight in the vicinity of the points of interſection with the axis, in order to agree with our obſervations which ſhow the moderate extenſions to be as the extending forces, the curve must be concave towards the axis in all its attractive branches, becauſe it cuts it again. Therefore the curve A *e' d'* of fig. 5. (n⁰ I.) muſt make a finite angle with AD or AB, and it muſt, in all probability, be alſo concave towards AD in the neighbourhood of *d'.* It may however be convex in ſome part of the intermediate arch. We have made ex­periments on the extenſions of different bodies, and find great diverſities in this reſpect : But in all, the moderate extenſions were as the forces, and this with great accuracy till the body took a ſett, and remained longer than formerly when the extending force was removed.

We muſt now remark, that this correction of the Galilean hypotheſis of equal forces was ſuggeſted by the bending’ which is obſerved in all bodies which are ſtrained tranſverſely. Becauſe they are bent, the fibres on the convex side have been extended. We cannot say in what proportion this ob­tains in the different fibres. Our moſt diſtinct notions of the internal equilibrium between the particles render it high­ly probable that their extenſion is proportional to their diſtance from that fibre which retains its former dimensions. But by whatever law this is regulated, we ſee plainly that the actions of the ſtretched fibres muſt follow the propor­tions of ſome function of this diſtance, and that therefore the relative ſtrength of a beam is in all caſes ſuſceptible of mathematical determination.

We alſo ſee an intimate connection between the ſtrain and the curvature. This ſuggeſted to the celebrated James Bernoulli the problem of the Elastic Curve;, *i. e.* the curve into which an extenſible rigid body will be bent by a tranſverſe ſtrain. His ſolution in the Acta *Lipsiae* 1694 and 1695 is a very beautiful ſpecimen of mathematical diſcuſſion ; and we recommend *it* to the peruſal of the curious reader. He will find *it* very perſpicuouſly treated in the firſt volume of his works, publiſhed after his death, where the wide ſteps which he had taken in his inveſtigation are explained so as to be eaſily comprehended. His nephew Dan. Bernoulli has given an elegant abridgment in the Peterſburg Memoirs for 1729. The problem is too in­tricate to be fully diſcuſſed in a work like ours ; but it is alſo too intimately connected with our preſent ſubject to be entirely omitted. We muſt content ourſelves with ſhowing the leading mechanical property of this curve, from which the mathematician may deduce all its geometrical properties.

When a bar of uniform depth and breadth, and of a given length, is bent into an arch of a circle, the extenſion of the