tation of the youngeſt life and twice the expectation of the oldeſt life increased by unity and twice the perpetuity. Multiply this difference by the value of an annuity certain for a time equal to twice the expectation of the oldeſt life ; and by twice the same expectation divide the product, reſerving the quotient.

From twice the perpetuity ſubtract the reſerved quotient, and multiply the remainder by the perpetuity increaſed by unity. This laſt product divided by twice the expectation of the youngeſt life, and then ſubtracted from the perpetuity, will be the required value.

When twice the expectation of the youngeſt life is great­er than twice the expectation of the oldeſt life increaſed by unity and twice the perpetuity, the reſerved quotient, inſtead of being ſubtracted from twice the perpetuity, muſt be add­ed to it, and the ſum, not the difference, multiplied by the perpetuity increaſed by unity.

*Example,* Let the joint lives propoſed be a female life aged 10, and a male life aged 15 ; and let the table of ob­ſervations be the Sweden table for lives in general, and the rate of intereſt 4 *per cent.* Twice the expectations of the two lives are 90.14 and 83.28.

Twice the expectation or the oldeſt life, increaſed by uni­ty, and twice the perpetuity, is 134.28, which leſſens by

90.14 (twice the expectation of the youngeſt life), leaves

44.14 for the reſerved remainder. This remainder multi­plied by 24.045 (the value of an annuity certain for 83.28 years), and the product divided by 83.28 (twice the expec­tation of the oldeſt liſe), gives 12.744, the quotient to be reſerved ; which ſubtracted from double the perpetuity, and the remainder (or 37.255) multiplied by the perpetuity in­creaſed by unity (or by 26) gives 968.630, which divided by 90.14 (twice the expectation of the youngeſt life) and the quotient ſubtracted from the perpetuity, we have 14.254 for the required value.

The value of an annuity certain, when the number of years is a whole number with a fraction added (as will be commonly the case) may be beſt computed in the following manner. In this example the number of years is 83.28. The value of an annuity certain for 83 years is 24.035. The ſame value for 84 years is 24.072. The difference be­tween theſe two values is 0.37 ; which difference multiplied by .28 (the fractional part oſ the number of years), and the product (.0103) added to the leaſt ot the two values, will give 24.045 the yalue for 83.28 years.

*General Rule.* Call the correct value (ſuppoſed to be computed for any rate of intereſt) the first value. Call the value deduced (by the preceding problems) from the expec­tations at the ſame rate of intereſt, the ſecond value. Call the value deduced from the expectations for any other rate of intereſt the third value.

Then the difference between the first and ſecond values added to or ſubtracted from the third value, juſt as the first is greater or leſs than the ſecond, will be the value at the rate of intereſt for which the third value has been deduced from the expectations.

The following examples will make this perfectly plain.

*Example* I. In the two laſt tables the correct values are given of two joint lives among mankind, at large, without diſtinguiſhing between males and females, according to the Sweden obſervations, reckoning intereſt at 4 *per cent.* Let it be required to find from theſe values the values at 3 *per cent.* and let the ages of the joint lives be ſuppoſed 10 and 10.

The correct value by Table IV. (reckoning intereſt at 4 *per cent.)* is 16.141. The expectation of a life aged 10 is 45.07. The value deduced from this expectation at 4 per *cent,* by Prob. II. is 14.539. The value deduced by the ſame problem from the ſame expectation at 3 *per cent.* is 16.808. The difference between the first and ſecond values is 1.602, which, added to the third value (the first being greater than the second), makes 18.410, the value required.

*Example* II. Let the value be required of a single male life aged 10, at .3 *per cent.* intereſt, from the correct value at 4 *per cents* according to the Sweden obſervations.

Firſt, or correct value at 4 *per cent.* (by Table III.) is 18.674. The expectation of a male life aged 10 is 43.94.

The ſecond value (or the value deduced from this expec­tation by Prob. I.) is 17.838.

The third value (or the value deduced from the same ex­pectation at 3 *per cent)* is 21.277.

The difference between the first and ſecond is .836; which (ſince the first is greater than the ſecond) muſt be added to the third ; and the ſum (that is, 22.113) will be the value required.

The third value at 5 *per cent.* is 15.286 ; and the diffe­rence added to 15.286 makes *16.122* the value of a male life aged 10 at 5 *per cent.* according to the Sweden obſerva­tions. The exact value at 5 *per cent.* is (by Table III.) 16.014.

Again : The difference between 16,014 (the correct va­lue at 5 *per cent.),* and 15.286 (the value at the ſame in­tereſt deduced from the expectation ), is .728 ; which, added (becauſe the first value is greater than the ſecond) to 13.335 (the value deduced at 6 *per cent,* from the expectation) gives 14.063, the value of the ſame life, reckoning intereſt at 6 *per cent.*

Theſe deductions, in the caſe of ſingle lives particularly, are ſo eaſy, and give the true values ſo nearly, that it will be ſcarcely ever neceſſary to calculate the exact values (ac­cording to any given obſervations) for more than one rate of intereſt,

If,· *for* inſtance, the correct values are computed at 4 *per cent.* according to any obſervations, the values at 3, 3 1/2, 4 1/2, 5, 6, 7, or 8 *per cent,* may be deduced from them by the preceding rules as occasion may require, without much la­bour or any danger of conſiderable errors. The values thus deduced will seldom differ from the true values ſo much as a tenth of a year’s purchase. They will not generally differ more than a 2c4h or 3cth of a year’s purchaſe. In joint lives they will differ leſs than in single lives, and they will come equally near to one another whatever the rates of in­tereſt are.

The preceding tables furniſh the means of determining the exact differences between the values of annuities, as they are made to depend on the ſurvivorſhip of any male or female lives ; which hitherto has been a desideratum *of* considerable conſequence in the doctrine, of life-annuities. What has made this of conſequence is chiefly the multitude of ſocieties lately establiſhed in this and foreign countries for providing annuities for widows. The general rule for calculating from theſe tables the value of ſuch annuities is the following.

*Rule.,* “ Find in Table III. the value of a female liſe at the age of the wife. From this value ſubtract the value in Table IV. of the joint continuance of two lives at the ages of the husband and wife. The remainder will be the value in a ſingle preſent payment of an annuity for the life of the wife, ſhould ſhe be left a widow. And this laſt value divi­ded by the value of the joint lives increaſed by unity, will be the value of the ſame annuity in annual payments during the joint lives, and to commence immediately.”

*Example.* Let the age of the wife be 24, and of the husband 30. The value in Table III. (reckoning intereſt at 4 *per cent.)* of a female life aged 24, is 17.252. The va­lue in Table IV. of two joint lives aged 24 and 30, is 13. 455, which ſubtracted from 17.252 leaves 3.797, the