It is as AI/bI, or as ER/CR. Suppoſe the figure to turn round the axis. I or R deſcribe circumferences of circles ; and the whole light paſſing through this circumference is as the circumference, or as the radius, and as the denſity jointly.

It is therefore as ER/CR × CR, that is, as ER. Draw any ſtraight line Em, cutting RN in *s,* and any other ordinate FL in xRs. The whole light which illuminates the cir­cumference deſcribed by I is to the whole light which il­luminates the centre *b* as ER to EC, or as Rs to *Cm.* In like manner, the whole light which illuminates the circum­ference described by the point f in the circle of diſperſion is to the whole light which illuminates the centre *b,* as Fx to *Cm.* The lines *Cm,* RS, Fx, are therefore proportional to the whole light which illuminates the correſponding cir­cumferences in the circle of diſperſion. Therefore the whole light which falls on the circle whoſe radius is bI, will be re­preſented by the trapezium in CRS ; and the whole light which falls on the ring deſcribed by IA, will be repreſented by the triangle E *s* R ; and ſo of any other portions.

By conſidering the figure, we ſee that the diſtribution of the light is exceedingly unequal. Round the margin it has no ſenſible denſity ; while its denſity in the very centre is incomparably greater than in any other point, being expressed by the aſymptote of a hyperbola. Alſo the circle de­ſcribed with the radius Ab/2 contains 3/4ths of the whole light. No wonder then that the confuſion cauſed by the mixture of theſe circles of diſperſion is leſs than one ſhould expect ; besides, it is evident that the moſt lively or impreſſive co­lours occupy the middle of the ſpectrum, and are there much denſer than the rest. The margin is covered with an illumination of deep red and violet, neither of which co­lours are brilliant. The margin will be of a dark claret co­lour. The centre revives all the colours, but in a propor­tion of intenſity greatly different from that in the common priſmatic ſpectrum, becauſe the radiant points L, p*, b, g,* &c. by which it is illuminated, are at ſuch different diſtances from it. It will be white ; but we apprehend not a pure white, being greatly overcharged with the middle colours.

Theſe conſiderations ſhow that the coloured fringes, which are obſerved to border very luminous objects ſeen on a dark ground through optical inſtruments, do not proceed from the object-glaſs of a teleſcope or microſcope, but from an improper conſtruction of the eye-glaſſes. The chroma­tic diſperſion would produce fringes of a different colour, when they produce any at all, and the colours would be differently diſpoſed. But this diſperſion by the object-glaſs can hardly produce any fringes: its effect is a general and almoſt uniform mixture of circles all over the field, which produces an uniform hazineſs, as if the object were viewed at an improper diſtance, or out of its focus, as we vulgarly expreſs it.

We may at preſent form a good gueſs at the limit which this cauſe puts to the performance of a teleſcope. A point of a very diſtant object is repreſented, in the picture formed by the object-glaſs, by a little circle, whoſe diameter is at leaſt th of the aperture of the object-glaſs, making a very full allowance for the ſuperior brilliancy and denſity of the central light. We look at this picture with a magnifying eye-glaſs. This magnifies the picture of the point. If it am­plify it to ſuch a degree as to make it an object individually diſtinguiſhable, the confuſion is then ſenſible. Now this can be computed. An object ſubtending one minute of a degree is diſtinguiſhed by the dulleſt eye, even although it be a dark object on a bright ground. Let us therefore suppoſe a teleſcope, the object-glaſs of which is of ſix feet focal diſtance, and one inch aperture. The diameter of the cir­cle of chromatic diſperſion will be 1/300th of an inch, which ſubtends at the centre of the object-glaſs an angle of about 91/2 seconds. This, when magnified ſix times by an eye-glaſs, would become a diſtinguiſhable object ; and a teleſcope of this length would be indiſtinct if it magnified more than six times, it a point were thus ſpread out into a ſpot or uniform intenſity. But the ſpot is much leſs intenſe about its mar­gin. It is found experimentally that a piece of engra­ving, having fine croſs hatches, is not ſenſibly indiſtinct till brought ſo far from the limits of perfectly diſtinct vision,that this indiſtinctneſs amounts to 6' or 5' in breadth.— Therefore ſuch a teleſcope will be ſenſibly diſtinct when it magnifies 36 times ; and this is very agreeable to experi­ence.

We come, in the ſecond place, to the more arduous taſk of aſcertaining the error ariſing from the ſpherical figure of the ſurfaces employed in optical inſtruments.— Suffice it to ſay, before we begin, that although geometers have exhi­bited other forms of lenſes which are totally exempt from this error, they cannot be executed by the artiſt ; and we are therefore reſtricted to the employment of ſpherical sur­faces.

Of all the determinations which have been given of ſpheri­cal aberration, that by Dr Smith, in his Optics, which is an improvement of the fundamental theorem of that moſt elegant geometer Huyghens, is the moſt perſpicuous and palpable. Some others are more conciſe, and much better fitted for after uſe, and will therefore be employed by us in the proſecution of this article. But they do not keep in view the optical facts, giving the mind a picture of the progreſs of the rays, which it can contemplate and diſcover amidſt many modifying circumſtances. By ingenious ſubſtitutions of analytical ſymbols, the inveſtigation is rendered expedi­tious, conciſe, and certain ; but theſe are not immediate ſymbols of things, but of operations of the mind ; objects ſufficiently ſubtile of themſelves, and having no need of ſubſtitutions to make us loſe ſight of the real ſubject ; and thus our occupation degenerates into a proceſs almoſt without ideas. We ſhall therefore ſet out with Dr Smith’s funda­mental Theorem.

I. *In Reflections.*

Let AVB (fig. 3.) be a concave ſpherical mirror, of which C is the centre, V the vertex, CV the axis, and F the focus of an infinitely ſlender pencil of parallel rays paſ­ſing through the centre. Let the ray aA, parallel to the axis, be reflected in AG, crossing the central ray CV in *f* Let AP be the sine of the ſemi-aperture AV, AD its tan­gent, and CD its ſecant.

The aberration Ff from the principal focus of central rays is equal to 1/2 of the excess VD of the ſecant above the radius, or very near equal to 1/2 of VP, the verſed sine of the ſemi-aperture.

For becauſe AD is perpendicular to CA, the points C, A, D, are in a circle, of which CD is the diameter ; and becauſe *Af* is equal to Cf*,* by reaſon of the equality of the angles fAC, fCA, and CA*a, f is* the centre of the circle through C, A, D, and fD is = 1/2CD. But FC is = *1/2CV.* Therefore Ff is 1/2 of VD.

But becauſe DV : VP = DC : VC, and DC is very little greater than VC when the aperture AB is moderate, DV is very little greater than VP, and *Εf* is very nearly equal to 1/2 of VP.

*Cor.* I. The longitudinal aberration is = AV2/4CV, for PV is very nearly = AV2/2CV.